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This document consists of 6 pages.

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ARMY SERVICE FORCES
UNITED STATES ENGINEER OFFICE
MANHATTAN DISTRICT
OAK RIDGE, TENNESSEE

IN REPLY REFER TO

EIDMG-21-a

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M-2487

28 April 1945

* 19970000461 *

Subject: Maximizing Output from S-50

MEMORANDUM to H. C. Vernon

1. The quantity $T_p(k-1)$ is a figure of merit that may be maximized by selecting definite operating conditions. Maximizing this quantity is equivalent to maximizing the product of production rate times difference in concentration from normal previously referred to as

$$(C_p - C_n)A = P$$

in the 12 March letter to C. N. Roberts.

2. The recent experimental data collected by the S-50 Operations and communicated to us has substantiated our theoretical deductions and provided a means of readily forecasting with confidence the production results of various methods of operation.

3. This memorandum presents the derivation and calculations to show those combinations of feed rates, draw off rates and compositions which yield a maximum for the figure of merit quoted. Curve PC 14-4 (attached) presents the results in graphical form.

4. The following notation is used:

C_p = Concentration of product

C_f = Concentration of feed (C_f = .714)

C_w = Concentration of waste

T_p = Grams per day of T of product

T_f = Grams per day of T of feed

T_w = Grams per day of T of waste

y = Natural logarithm of maximum separation

H = Experimental constant grams T per column day

$V_p = \frac{T_p}{H}$ = rate of product in units of H

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$V_f = \frac{T_f}{H}$ = rate of feed in units of H

$V_w = \frac{T_w}{H}$ = rate of waste in units of H

5. Formulae:

$$(1) \quad \frac{C_p}{C_w} = \frac{1 + V_p}{V_p + e - (1 + V_p)y} = G(V_p, y)$$

This formula has been theoretically derived assuming continuous feed and continuous removal of product and waste. Comparison with experimental and operational data shows that it provides a satisfactory approximation for the actual intermittent process used at S-50. By taking the experimental constants

$$(2) \quad \begin{aligned} y &= 0.35 \\ H &= 36.44 \text{ grams per column per day} \end{aligned}$$

a good fit is obtained for the relation obtained at S-50 between product concentration C_p and rate of product T_p (including a time attainment factor corresponding to 83 kg. T/day at 0.86%) for fixed $C_w = 0.95 \times 0.714 = 0.6783\%$. This fit is demonstrated in graph PC 14-2.

In addition to 1 there are the material balance relations:

$$(3) \quad C_f T_f = C_w T_w + C_p T_p$$

$$(4) \quad T_f = T_w + T_p$$

6. Derivation of Maximum:

To simplify problem formulae will be expressed in terms of:

$$(5) \quad V_p = \frac{T_p}{H}, \quad V_f = \frac{T_f}{H} \qquad V_w = \frac{T_w}{H}$$

Since H is an experimental constant with present value

(6) $H = 36.44$ (for an average of 83 kg. T/day at 0.86%)

the problem of maximizing

$$(7) \quad E = T_p (k-1)$$

is equivalent to maximizing

$$(8) \quad F = \frac{E}{H} = \frac{T_p (k-1)}{H}$$

$$= V_p (k-1) \qquad k = C_p/C_f$$

By (3) and (4), dividing each equation by H

(9) $$C_f V_f = C_w V_w + C_p V_p$$

(10) $$V_f = V_w + V_p$$

so that by eliminating V_w

(11) $$C_f V_f = C_w (V_f - V_p) + C_p V_p$$

Hence by 11

(12) $$k - 1 = \frac{C_f}{C_p} = \frac{C_w}{C_p}\left(1 - \frac{V_p}{V_f}\right) + \frac{V_p}{V_f}$$

But by (1)

(13) $$\frac{C_p}{C_w} = \frac{1 + V_p}{V_p + s - (1+V_p)y} = G(V_p, y)$$

so that

(14) $$k = \left\{\left[G(V_p,y)\right]^{-1}\left(1 - \frac{V_p}{V_f}\right) + \frac{V_p}{V_f}\right\}^{-1}$$

$$= \frac{G(V_p,y)}{1 - \frac{V_p}{V_f} + \frac{V_p}{V_f} G(V_p,y)}$$

while

(15) $$k - 1 = \frac{\left(1 - \frac{V_p}{V_f}\right)\left[G(V_p,y) - 1\right]}{1 - \frac{V_p}{V_f} + \frac{V_p}{V_f} G(V_p,y)}$$

$$= \frac{G(V_p,y) - 1}{1 + \frac{V_p}{V_f - V_p} G(V_p,y)}$$

The functions to be maximized are thus by 3

(16) $$F = V_p (k - 1)$$

$$= \frac{V_p \left[G(V_p,y) - 1\right]}{1 + \frac{V_p}{V_f - V_p} \; G(V_p,y)}$$

Since by (1)

$$(17) \qquad G(V_p,y) = \frac{1 + V_p}{V_p + e^{-(1+V_p)y}}$$

If the observed value $y = 0.35$, then there are only two variables V_p and V_f : i.e, product rate and feed rate in units H that can be varied. Maximum values of F can be found for every fixed rate of feed V_f, for if

$$V_p = 0 \qquad \text{then} \qquad F = 0$$

$$V_p = V_f \qquad \text{then} \qquad F = 0$$

hence there exists a maximizing value of V_p between $V_p = 0$ and $V_p = V_f$.

Summary of formulae

For computing maximum

$$(18) \qquad F = V_p\,(k - 1) = \frac{V_p \left[G(V_p,y) - 1\right]}{1 + \frac{V_p}{V_f - V_p} \; G(V_p,y)} \qquad \left(= \frac{T_p}{H}\,(k-1)\right)$$

where

$$G(V_p,y) = \frac{1 + V_p}{V_p + e^{-(1+V_p)y}}$$

Graphs of F will be plotted for $y = 0.35$ and

$$(19) \qquad V_f = 5, 10, 15, 20$$

From the values of F and V_p, the relations

$$(20) \qquad k = \frac{F}{V_p} + 1 = \frac{C_p}{C_o} \qquad C_o = .714$$

may be used to find C_p, and the relation

$$(21) \qquad \frac{C_p}{C_w} = G(V_p,y)$$

may then be employed to find G_w.

In the numerical case since the unit of flow is H, the only assigned constant is $y = 0.35$.

AUXILIARY TABLE

$y = 0.35$

V_p	$G(V_p,y)$	$V_p\left[G(V_p,y)-1\right]$
0	1.4190	.0000
.5	1.3742	.1871
1.0	1.3363	.3363
1.5	1.3041	.4561
2.0	1.2768	.5536
2.5	1.2528	.6320
3	1.2320	.6960
4	1.1979	.7816
5	1.1713	.8565
6	1.1501	.9006
7	1.1330	.9310

AUXILIARY TABLE OF $\frac{V_p}{V_f-V_p}$ $G(V_p,y)$

V_p \ V_f	5	10	15	20
0	.0000	.0000	.0000	.0000
.5	.1537	.07233	.04739	.03524
1.0	.3341	.1485	.09545	.07033
1.5	.5589	.2301	.1449	.10574
2	.8512	.3192	.1964	.14186
2.5	1.2528	.4176	.2506	.17897
3.0	1.8480	.5280	.3080	.2174
4	4.7916	.7986	.4356	.29947
5	∞	1.1713	.5856	.39043
6	∞	1.7251	.76673	.49290
7	∞	2.6444	.99137	.61008

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TABLE OF $k = \frac{F}{V_p} + 1$

V_f

V_p	5	10	15	20	∞
0					
.5	1.3244	1.3490	1.3572	1.3614	1.3742
1.0	1.2521	1.2928	1.3070	1.3142	1.3363
2.0	1.1495	1.2098	1.2313	1.2424	1.2768
3.0	1.0814	1.1518	1.1774	1.1908	1.2320
4	1.0337	1.1086	1.1361	1.1504	1.1954
5		1.07890	1.1080	1.1203	1.1713
6		1.0551	1.0849	1.1005	1.1501
7		1.0365	1.0668	1.0826	1.1330

TABLE OF F

$$F = \frac{V_p\left[G(V_p,y)-1\right]}{1 + \frac{V_p}{V_f-V_p}\,G(V_p,y)}$$

V_p	5	10	15	20	∞
0	0	0	0	0	.0000
.5	.1622	.1745	.1786	.1807	.1871
1.0	.2521	.2928	.3070	.3142	.3363
1.5	.2926	.3708	.3983	.4125	.4561
2.0	.29905	.4196	.4627	.4848	.5536
2.5	.28053	.4458	.5053	.5360	.6320
3.0	.24438	.4555	.5321	.5717	.6960
4	.1349	.4345	.5444	.6015	.7816
5	0	.3945	.5402	.6160	.8565
6		.3305	.5098	.60325	.9006
7		.2555	.4675	.5782	.9310

Walter Bartky

WALTER BARTKY per/ DJ

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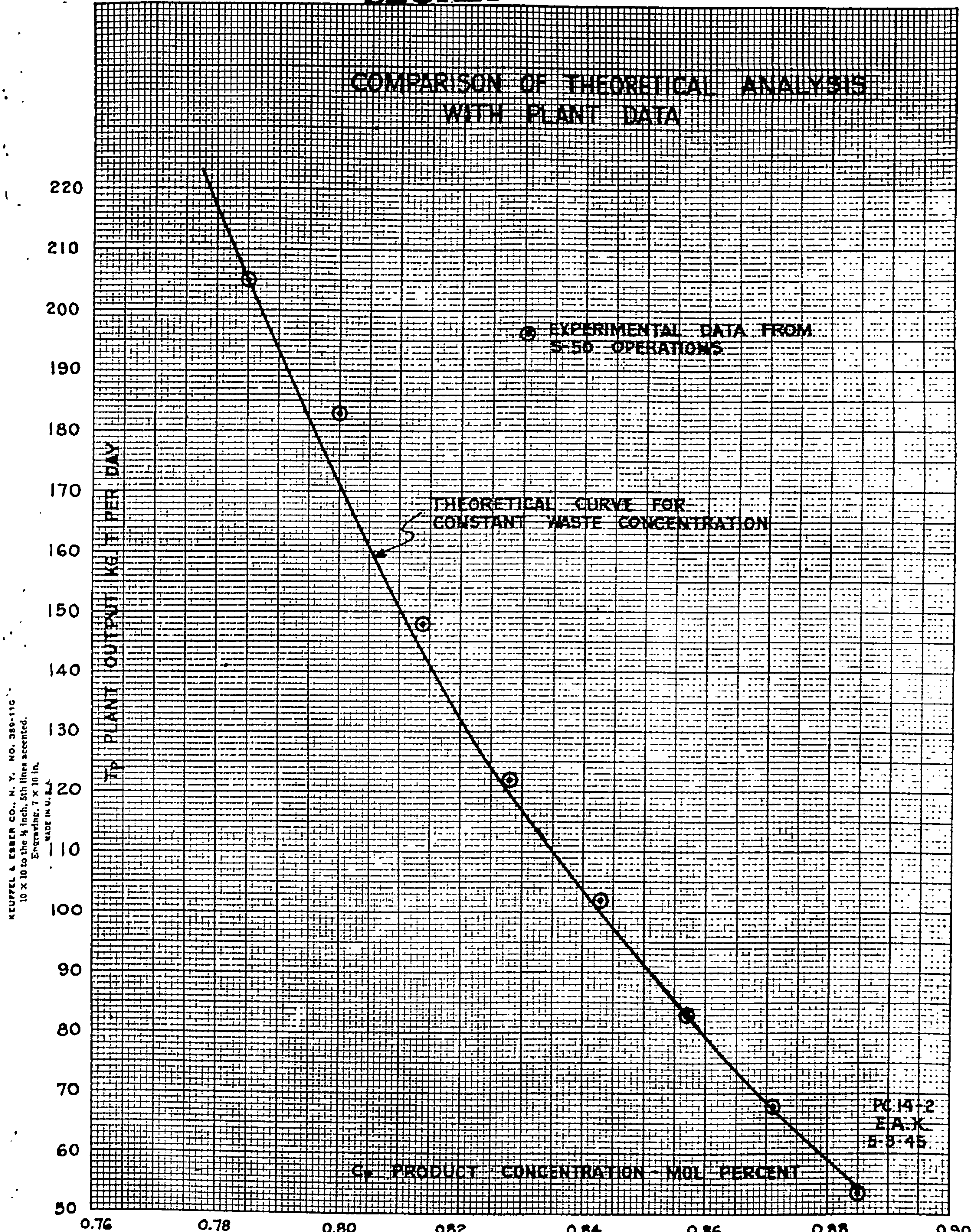

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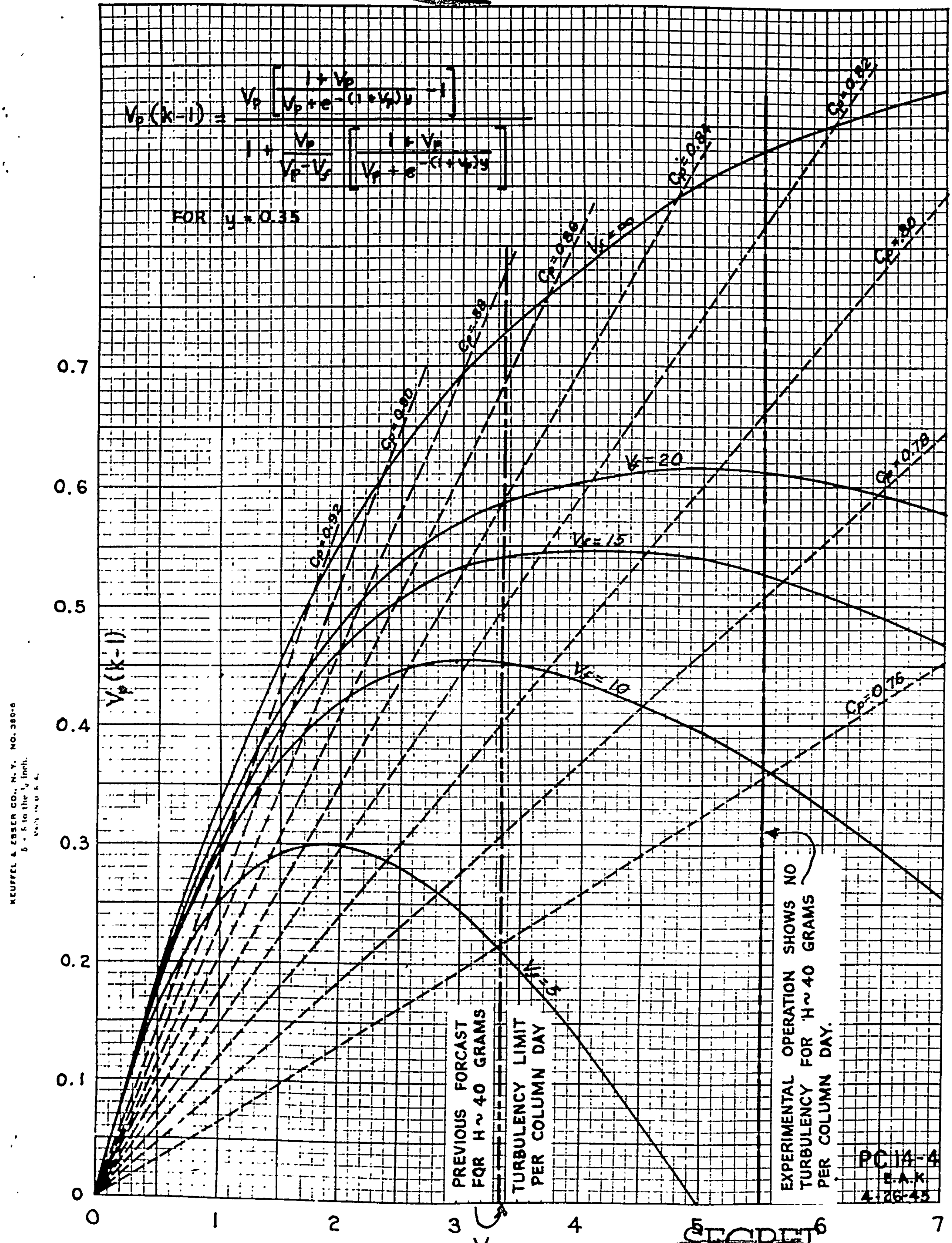

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