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## A Graphical Determination of the Radionuclide Inventory in the Concentrate and Tailings from Processing Facilities

D. J. Crawford

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Health and Safety Research Division

A GRAPHICAL DETERMINATION OF THE RADIONUCLIDE INVENTORY IN  
THE CONCENTRATE AND TAILINGS FROM PROCESSING FACILITIES

D. J. Crawford <sup>dy</sup>

Date Published: October 1979

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# A GRAPHICAL DETERMINATION OF THE RADIONUCLIDE INVENTORY IN THE CONCENTRATE AND TAILINGS FROM PROCESSING FACILITIES

D. J. Crawford

## ABSTRACT

During radiological surveys of radionuclide processing facilities, one must make a determination of the expected inventory of all members of the three naturally occurring radionuclide decay chains. A set of graphs has been developed that display the decay and ingrowth of all members of the  $^{232}\text{Th}$ ,  $^{238}\text{U}$ , and  $^{235}\text{U}$  decay series. These graphs may be used in the field to predict present inventories from a knowledge of previous inventories. Equations used to generate several of the graphs are given in tables.

## I. INTRODUCTION

Natural ores have been in residence in the earth's crust for millions of years and, as a result, contain not only the parent radionuclides of the uranium, actinium, and/or thorium series, but approximately equilibrium activities of all daughters. (Sill<sup>1</sup> has shown that this state of equilibrium exists in various degrees both in the original ore and in tailings piles.) The ultimate goal of milling processes is usually the extraction and refinement of quantities of  $^{232}\text{Th}$ ,  $^{238}\text{U}$ ,  $^{235}\text{U}$ , or  $^{226}\text{Ra}$  from these ores. The remainder of the processed material, or residues, retains those radionuclides in the decay chains that were not removed with the finished product. These residues are typically disposed of either at the processing facility or at remote locations.

Radiological assessments of land areas used for the storage of either wastes or finished products require some knowledge of the relative activity of each member of the three radioactive decay chains. Available information usually consists of only an estimate of the relative activities

at the time of separation or soon thereafter. It is possible to calculate the activity of each chain member at an arbitrary time,  $t$ , from this information through the use of the Bateman<sup>2</sup> equations. Due to the complexity of these equations, it would obviously be desirable to possess a set of normalized graphs that contain all of the information inherent in these relations. The author has developed such a set of graphs for the members of the three naturally occurring decay chains. These graphs may be used in the radiological assessment of a contaminated facility whenever a knowledge of the state of contamination at some earlier time is available. An assumption has been made that the chain members do not undergo further separation by such natural processes as leaching.

## II. DEFINING EQUATIONS

The differential equation whose solution yields the number of radioactive daughter atoms,  $N_2$ , as a function of time is

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t) .$$

In this relation,  $N_1(t)$  is the number of parent atoms present at time,  $t$ , and  $\lambda_1$  and  $\lambda_2$  are the decay constants of parent and daughter, respectively. The decay constant may be determined from the relation

$$\lambda = \ln 2 / T_{1/2} ,$$

where  $T_{1/2}$  is the radiological half-life.

Bateman solved this equation for the  $n^{\text{th}}$  member of a radioactive chain<sup>2</sup> and has given a solution of

$$N_n(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + \dots + C_n e^{-\lambda_n t} = \sum_{n=1}^N C_n e^{-\lambda_n t},$$

where  $N_n(t)$  is the number of atoms of the  $n^{\text{th}}$  member present at time  $t$  and

$$C_n = \frac{\lambda_1 \lambda_2 \dots \lambda_n}{(\lambda_1 - \lambda_n)(\lambda_2 - \lambda_n) \dots (\lambda_{n-1} - \lambda_n)} N_i.$$

The decay constant,  $\lambda_n$ , has been described previously, and  $N_i$  is the initial number of atoms of the parent radionuclide. The activity of the  $n^{\text{th}}$  member is given by  $\lambda_n N_n$ . By direct substitution of values of  $\lambda$  obtained from Kocher<sup>3</sup>, the equations in Tables 1, 2, and 3 were generated. These equations assume an initially pure sample of the parent radionuclide with an initial activity of  $A_0$  curies. While the set of graphs contains plots of other possible initial inventories, the equations that describe these plots are developed in a manner analogous to Tables 1, 2, and 3 and are not included here. In these equations,  $\lambda$  is in units of years<sup>-1</sup> and  $t$  is in years. Any term that is small enough to be neglected has been designated by the symbol  $\Delta$ .

### III. DESCRIPTION OF THE GRAPHS

Figure 1 is a plot of the decay of an initial quantity,  $A_0$ , of  $^{232}\text{Th}$  and the ingrowth of the  $^{228}\text{Ra}$ ,  $^{228}\text{Ac}$ ,  $^{228}\text{Th}$ , and  $^{224}\text{Ra}$  daughters. It assumes an initially pure  $^{232}\text{Th}$  sample. As has been mentioned, it is rare that a pure sample of  $^{232}\text{Th}$  will be found at a survey site. Currently employed separation processes extract  $^{230}\text{Th}$  and  $^{228}\text{Th}$  along with  $^{232}\text{Th}$ . Such circumstances may be accounted for by the use of several of the graphs, which will be introduced in subsequent paragraphs

and by the use of the insert to Fig. 1. This insert is a plot as a function of time of the quantity of  $^{228}\text{Th}$  remaining from an initial quantity,  $A_0$ . Also shown is the ingrowth of the  $^{224}\text{Ra}$  daughter of  $^{228}\text{Th}$ . It will be noticed that  $^{224}\text{Ra}$  grows into equilibrium very quickly and may be assumed (in most realistic cases) to be in equilibrium with the remainder of the initial  $^{228}\text{Th}$ .

Figure 2 is a plot of the decay of an initially pure sample of  $^{238}\text{U}$  as a function of the time since separation. The short-lived daughters  $^{234}\text{Th}$  and  $^{234m}\text{Pa}$  are shown on this figure and the insert is a plot of the decay of any  $^{234}\text{U}$  that was extracted along with the  $^{238}\text{U}$  as well as the ingrowth of the  $^{230}\text{Th}$  and  $^{226}\text{Ra}$  daughters, which arise from this initial  $^{234}\text{U}$ . The quantity of  $^{234}\text{U}$ ,  $^{230}\text{Th}$ , and  $^{226}\text{Ra}$  that arises due to the initial  $^{238}\text{U}$  is displayed in Fig. 3 as a function of time following separation. These daughters appear on a time scale that is much larger than the  $^{234}\text{Th}$  and  $^{234m}\text{Pa}$  daughters since  $^{234}\text{U}$  grows into equilibrium with  $^{238}\text{U}$  at a much slower rate. Inserts A and B are plots of the decay of any initial  $^{230}\text{Th}$ , which may have been present, and of the ingrowth of  $^{226}\text{Ra}$  arising from this  $^{230}\text{Th}$ .

Figure 4 is a plot of the quantity of  $^{235}\text{U}$  as a function of time following separation. Also, shown is the ingrowth of the  $^{231}\text{Th}$  daughter, while the insert is a plot of the growth of the  $^{231}\text{Pa}$ ,  $^{227}\text{Ac}$ , and  $^{223}\text{Ra}$  daughters. The activity of  $^{227}\text{Th}$  and  $^{223}\text{Fr}$  may be approximated by multiplying the value for  $^{227}\text{Ac}$  by the constants 0.986 and 0.014, respectively.

Upon removal of only the parents of the three natural chains, the remaining "tailings" activity consists of the daughter products as well as any parents that are not extracted. If the ore has been in

existence long enough to allow for equilibrium of all members of the decay series, the tailings will contain equilibrium activities of all daughters. The resultant activity of a particular daughter at any time following separation is the sum of two contributions. There will exist the remainder of the initial daughter activity, having decayed to a value given by the relation

$$A(t) = A_0 e^{-\lambda t},$$

where

$A(t)$  = activity at time  $t$ ,

$A_0$  = activity of the daughter at  $t = 0$ ,

$\lambda$  = decay constant for the daughter.

There will also be a growth of new daughter activity due to the decay of the initial inventory of each radionuclide, which is a precursor to the daughter under study.

As an example, consider the activity of  $^{228}\text{Th}$  that exists in a sample containing all of the daughters of  $^{232}\text{Th}$ . The initial  $^{228}\text{Th}$  activity will decay with a radioactive half-life of 1.91 years. At the same time, there will be an ingrowth of new  $^{228}\text{Th}$  activity from the decay of the initial quantities of  $^{228}\text{Ac}$  and  $^{228}\text{Ra}$ . All three contributions must be accounted for in determining the total activity. Figures 5, 6, and 7 are plots as a function of time of the total activity of all daughters of the three natural decay chains. These graphs assume that at time  $t = 0$ , the activities of all daughters were the same,  $A_0$ . In addition, the graphs assume complete removal of the chain parent. As in the first four graphs (Figs. 1-4), convenient time scales have been chosen in order to display the entire history of each daughter.

Since it has been assumed that all daughters exist with the same activity,  $A_0$ , at  $t = 0$ , several of the graphs do not apply to the currently realistic case in which  $^{234}\text{U}$  is extracted along with the  $^{238}\text{U}$  or when  $^{228}\text{Th}$  comes out with  $^{232}\text{Th}$ . In these cases, the activities of  $^{234}\text{U}$  and  $^{228}\text{Th}$  at  $t = 0$  are depressed below the values of other daughters. As a result, the actual amount of each daughter below  $^{234}\text{U}$  and  $^{228}\text{Th}$  will be less than the amount predicted assuming complete equilibrium of all daughters following separation. Those graphs that are affected are shown as dashed lines and may only be employed in cases where the activities of all daughters of the three chains were equal at time  $t = 0$ . There is no effect on the activity of any daughter that occurs before  $^{234}\text{U}$  and  $^{228}\text{Th}$ , and the solid graphs may be used in either case. By way of example, consider the inventory of  $^{224}\text{Ra}$  in a tailings pile that initially contained  $A_0$  curies each of  $^{228}\text{Ra}$ ,  $^{228}\text{Ac}$ , and  $^{224}\text{Ra}$ , and  $A_1$  curies of  $^{228}\text{Th}$ . Figure 5 shows the ingrowth of  $^{228}\text{Th}$  and  $^{224}\text{Ra}$  from the initial quantity of  $^{228}\text{Ra}$ . If we find a value  $f$  for a given time after separation, the activity of  $^{224}\text{Ra}$  due to the initial  $^{228}\text{Ra}$  is  $A_0 f$ . Due to its short half-life compared to the next member in the decay series,  $^{228}\text{Ac}$  ( $^{228}\text{Th}$ ) contributes a negligible amount to the ingrowth of new  $^{228}\text{Th}$  and  $^{224}\text{Ra}$ . The growth of  $^{224}\text{Ra}$  due to the initial activity of  $A_1$  curies of  $^{228}\text{Th}$  may be found in the insert to Fig. 1. For a given time, a value  $f$  may be found from this graph and the activity of  $^{224}\text{Ra}$  from the initial  $^{228}\text{Th}$  computed by  $A_1 f$ . Finally, the decay of  $^{224}\text{Ra}$  due to the initial  $^{224}\text{Ra}$  is shown in the insert to Fig. 5. The total activity of  $^{224}\text{Ra}$  is now the sum of the three quantities determined above. If  $^{228}\text{Th}$  also had been in an abundance of  $A_0$  curies, the inventory of  $^{224}\text{Ra}$  could have been read directly from graph C, Fig. 5. In this case, a factor  $f$  would be found and multiplied by  $A_0$  curies.

As was mentioned previously, all extraction processes currently in use remove isotopes of the parent along with the parent itself. If  $x$  percent of the  $^{238}\text{U}$  is removed from the original ore, then  $x$  percent of  $^{234}\text{U}$  and  $^{235}\text{U}$  will also be removed. The result is that the inventory of all radionuclides in both the finished product and tailings will depend on the value of  $x$ . Typical extraction percentages are 85, 90, and 95%. Figures 8 through 20 give the activity of each member of the three natural chains for all three extraction fractions. These graphs are suggested for use in the field when the extraction fraction is known. If the fraction is not known, it is generally assumed to be 90%.<sup>4</sup> They are much simpler to use than the general graphs since the activity of any radionuclide, in either the finished product or waste, may be read as a single value from one of the graphs. All values are given as percent of the initial parent activity in the non-separated ore. As an example, consider an original ore with  $A_0$  curies of  $^{232}\text{Th}$  from which 90% of the  $^{232}\text{Th}$  and  $^{228}\text{Th}$  are removed. Figure 9 shows the activity of all daughters in the finished product as a function of the time following separation. Setting the time equal to 20 years, the activity of  $^{228}\text{Th}$  and  $^{224}\text{Ra}$  may be seen to be approximately 0.78A. Figure 12 gives the activity of all daughters in the waste material.

#### IV. USE OF THE GENERAL GRAPHS

The use of the graphs may best be illustrated by an example; the figures are taken from a typical processing plant. From May 1951 to February 1964, the plant processed 1.7 million tons of ore with an average grade of 0.32%  $\text{U}_3\text{O}_8$ . This produced 4787 tons of  $\text{U}_3\text{O}_8$ . The steps

required to convert all weights to activity in curies are straightforward and will not be explained here. The assumption is made that all daughters of both  $^{238}\text{U}$  and  $^{235}\text{U}$  are present in equilibrium before processing and that  $^{234}\text{U}$  and  $^{235}\text{U}$  are carried along with  $^{238}\text{U}$  in the concentrate.

These assumptions lead to the results shown in Table 4. As an example, one may determine the activity of the  $^{226}\text{Ra}$  daughter at a given time. In the concentrate,  $^{226}\text{Ra}$  grows both as a daughter of the initial  $^{238}\text{U}$  and as a daughter of the initial  $^{234}\text{U}$ . Due to the extremely slow ingrowth of  $^{226}\text{Ra}$ , little will exist until several thousands of years. For the purposes of this example, set the time equal to  $10^5$  years after separation. Figure 3 shows the  $^{226}\text{Ra}$  activity that results from the initial  $^{238}\text{U}$  to be  $(1203)(0.085) = 102.3$  Ci at time  $t = 10^5$  years. The insert to Fig. 2 shows the ingrowth of  $^{226}\text{Ra}$  from the initial  $^{234}\text{U}$ . Setting  $t = 10^5$  years, a value of  $(1203)(0.49) = 589.5$  Ci is obtained. The total activity of  $^{226}\text{Ra}$  in the concentrate is then  $(102.3 + 589.5) = 691.8$  Ci.

Turning now to the tailings, or waste products, it is noted that  $^{226}\text{Ra}$  has several sources of production. It is a daughter of the remaining  $^{238}\text{U}$  and  $^{234}\text{U}$ , as well as a daughter of any  $^{234}\text{Th}$ ,  $^{234m}\text{Pa}$ , and  $^{230}\text{Th}$ . Due to extremely small half-lives of  $^{234}\text{Th}$  and  $^{234m}\text{Pa}$  relative to  $^{226}\text{Ra}$ , these two daughters may be neglected with little error. The sources of  $^{226}\text{Ra}$  now consist of the decay of  $^{238}\text{U}$ ,  $^{234}\text{U}$ , and  $^{230}\text{Th}$ , as well as the remainder of the initial  $^{226}\text{Ra}$ .

Figure 3 gives the growth of  $^{226}\text{Ra}$  from  $^{238}\text{U}$ , and looking at  $t = 10^5$  years, one finds a value of  $(177)(0.085) = 15$  Ci. The growth of  $^{226}\text{Ra}$  from the initial  $^{230}\text{Th}$  is shown in the insert to Fig. 3. With  $t = 10^5$  years, a value of  $(1380)(0.42) = 579.6$  Ci is obtained. Finally, the decay of the initial  $^{226}\text{Ra}$  is shown in the insert to Fig. 6. With

$t = 10^5$  years, essentially all of the initial  $^{226}\text{Ra}$  has decayed. Therefore, the total activity of  $^{226}\text{Ra}$  in the tailings pile after  $10^5$  years is  $15 + 86.7 + 579.6 = 681.3$  Ci.

If all of the  $^{234}\text{U}$  remained in the tailings, such that all daughter activities were the same after separation, the value would be found directly from the insert to Fig. 6. Setting  $t = 10^5$  years, one reads a value of  $(1380)(0.91) = 1256$  Ci. Note that use has been made of the dashed curve. This has assumed an initial  $^{238}\text{U}$  fraction of zero. With an initial activity of 177 Ci of  $^{238}\text{U}$ , an extra 15 Ci of  $^{226}\text{Ra}$  would be present, as was shown in the preceding paragraph. Therefore, the total activity will be  $1256 + 15 = 1271$  Ci of  $^{226}\text{Ra}$  after  $10^5$  years, a drop of only 109 Ci.

It will be noted from Table 4 that roughly 87% of the  $^{238}\text{U}$ ,  $^{234}\text{U}$ , and  $^{235}\text{U}$  were removed by the extraction process. Since this fraction is known, it is much easier to use Figs. 14 and 16 and read the values directly from these graphs. Setting  $t = 10^5$  years, the amount of  $^{226}\text{Ra}$  in the product may be read directly from Fig. 14 and will be noted as being given by  $(0.50)(1380) = 690$  Ci. The amount of  $^{226}\text{Ra}$  in the tailings may be found from Fig. 16, and setting  $t = 10^5$  years, it is found that the activity is equal to  $(0.495)(1380) = 683$  Ci.

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Table 1. General equations for the growth of the  $^{238}\text{U}$  decay series

$$\text{Activity } ^{238}\text{U} = A_0 e^{-1.551 \times 10^{-10} t}$$

$$\text{Activity } ^{234}\text{Th} = A_0 (e^{-1.551 \times 10^{-10} t} - e^{-10.50 t})$$

$$\text{Activity } ^{234m}\text{Pa} = A_0 (e^{-1.551 \times 10^{-10} t} - e^{-10.50 t} \\ + \Delta 3.38 \times 10^{-5} e^{-3.1 \times 10^5 t})$$

$$\text{Activity } ^{234}\text{U} = A_0 (e^{-1.551 \times 10^{-10} t} + \Delta 2.66 \times 10^{-7} e^{-10.50 t} \\ - \Delta 3.05 \times 10^{-16} e^{-3.1 \times 10^5 t} - e^{-2.8 \times 10^{-6} t})$$

$$\text{Activity } ^{230}\text{Th} = A_0 (e^{-1.551 \times 10^{-10} t} - \Delta 2.2 \times 10^{-13} e^{-10.50 t} \\ + \Delta 8.5 \times 10^{-24} e^{-3.1 \times 10^5 t} - 1.47 e^{-2.8 \times 10^{-6} t} \\ + 0.477 e^{-9.0 \times 10^{-6} t})$$

$$\text{Activity } ^{226}\text{Ra} = A_0 (e^{-1.551 \times 10^{-10} t} + \Delta 9.1 \times 10^{-18} e^{-10.50 t} \\ - \Delta 1.19 \times 10^{-35} e^{-3.1 \times 10^5 t} - 1.49 e^{-2.8 \times 10^{-6} t})$$

Table 1. (continued)

---

$$+ 0.48 e^{-9.0 \times 10^{-6}t} - 1.32 \times 10^{-4} e^{-4.33 \times 10^{-4}t}$$

---

$A_0$  = initial activity of  $^{238}\text{U}$ .

$t$  = in years.

Table 2. General equations for the growth of the  $^{232}\text{Th}$  decay series

---


$$\text{Activity } ^{232}\text{Th} = A_0 e^{-4.932 \times 10^{-11}t}$$

$$\text{Activity } ^{228}\text{Ra} = A_0 (e^{-4.932 \times 10^{-11}t} - e^{-1.205 \times 10^{-1}t})$$

$$\begin{aligned} \text{Activity } ^{228}\text{Ac} = A_0 (e^{-4.932 \times 10^{-11}t} - e^{-1.205 \times 10^{-1}t} \\ + \Delta 1.04 \times 10^{-4} e^{-990.3t}) \end{aligned}$$

$$\begin{aligned} \text{Activity } ^{228}\text{Th} = A_0 (e^{-4.932 \times 10^{-11}t} - 1.6e^{-1.205 \times 10^{-1}t} \\ - \Delta 3.83 \times 10^{-8} e^{-990.3t} + 0.35e^{-0.3628t}) \end{aligned}$$

$$\begin{aligned} \text{Activity } ^{224}\text{Ra} = A_0 (e^{-4.932 \times 10^{-11}t} - 1.6e^{-1.205 \times 10^{-1}t} \\ + \Delta 2.89 \times 10^{-9} e^{-990.3t} + 0.352e^{-0.3628t} \\ - \Delta 8.4 \times 10^{-6} e^{-69.11t}) \end{aligned}$$


---

$A_0$  = initial activity of  $^{232}\text{Th}$ .

Table 3. General equations for the growth of the  $^{235}\text{U}$  decay series

$$\text{Activity } ^{235}\text{U} = A_0 e^{-9.76 \times 10^{-10}t}$$

$$\text{Activity } ^{231}\text{Th} = A_0 (e^{-9.76 \times 10^{-10}t} - e^{-238t})$$

$$\text{Activity } ^{231}\text{Pa} = A_0 (e^{-9.76 \times 10^{-10}t} + 8.95 \times 10^{-8} e^{-238t} \\ - e^{-2.13 \times 10^{-5}t})$$

$$\text{Activity } ^{227}\text{Ac} = A_0 (e^{-9.76 \times 10^{-10}t} - 1.2 \times 10^{-11} e^{-238t} \\ - e^{-2.13 \times 10^{-5}t} + \Delta 6.65 \times 10^{-4} e^{-3.2 \times 10^{-2}t})$$

$$\text{Activity } ^{227}\text{Th} = A_0 (e^{-9.76 \times 10^{-10}t} - \Delta 7.37 \times 10^{-13} e^{-238t} \\ - e^{-2.13 \times 10^{-5}t} + \Delta 6.75 \times 10^{-4} e^{-3.18 \times 10^{-2}t} \\ + \Delta 3.71 \times 10^{-9} e^{-13.51t})$$

$$\text{Activity } ^{223}\text{Ra} = A_0 (e^{-9.76 \times 10^{-10}t} - \Delta 7.53 \times 10^{-14} e^{-238t}$$

Table 3. (continued)

---

$$- e^{-2.13 \times 10^{-5}t} + \Delta_6.73 \times 10^{-4} e^{-3.18 \times 10^{-2}t}$$

$$- \Delta_1 \times 10^{-8} e^{-13.51t} + \Delta_3 \times 10^{-9} e^{-22.12t}$$

---

$A_0$  = initial activity of  $^{235}\text{U}$ .

Table 4. Inventory of radionuclides from the uranium and actinium decay series

---

Inventory before processing	
Activity	$^{238}\text{U} = ^{234}\text{U} = ^{234}\text{Th} = ^{234m}\text{Pa} = ^{230}\text{Th} = ^{226}\text{Ra} = 1380 \text{ Ci}$
Activity	$^{235}\text{U} = ^{231}\text{Th} = ^{231}\text{Pa} = ^{227}\text{Ac} = ^{227}\text{Th} = ^{223}\text{Ra} = 63.3 \text{ Ci}$

---

Inventory after processing at 87% efficiency (concentrate)	
Activity	$^{238}\text{U} - ^{234}\text{U} = 1203 \text{ Ci}$
	$^{235}\text{U} = 55.2 \text{ Ci}$
(tailings)	
Activity	$^{238}\text{U} = ^{234}\text{U} = 177 \text{ Ci}$
	$^{235}\text{U} = 8.1 \text{ Ci}$
All daughter activities remain the same.	

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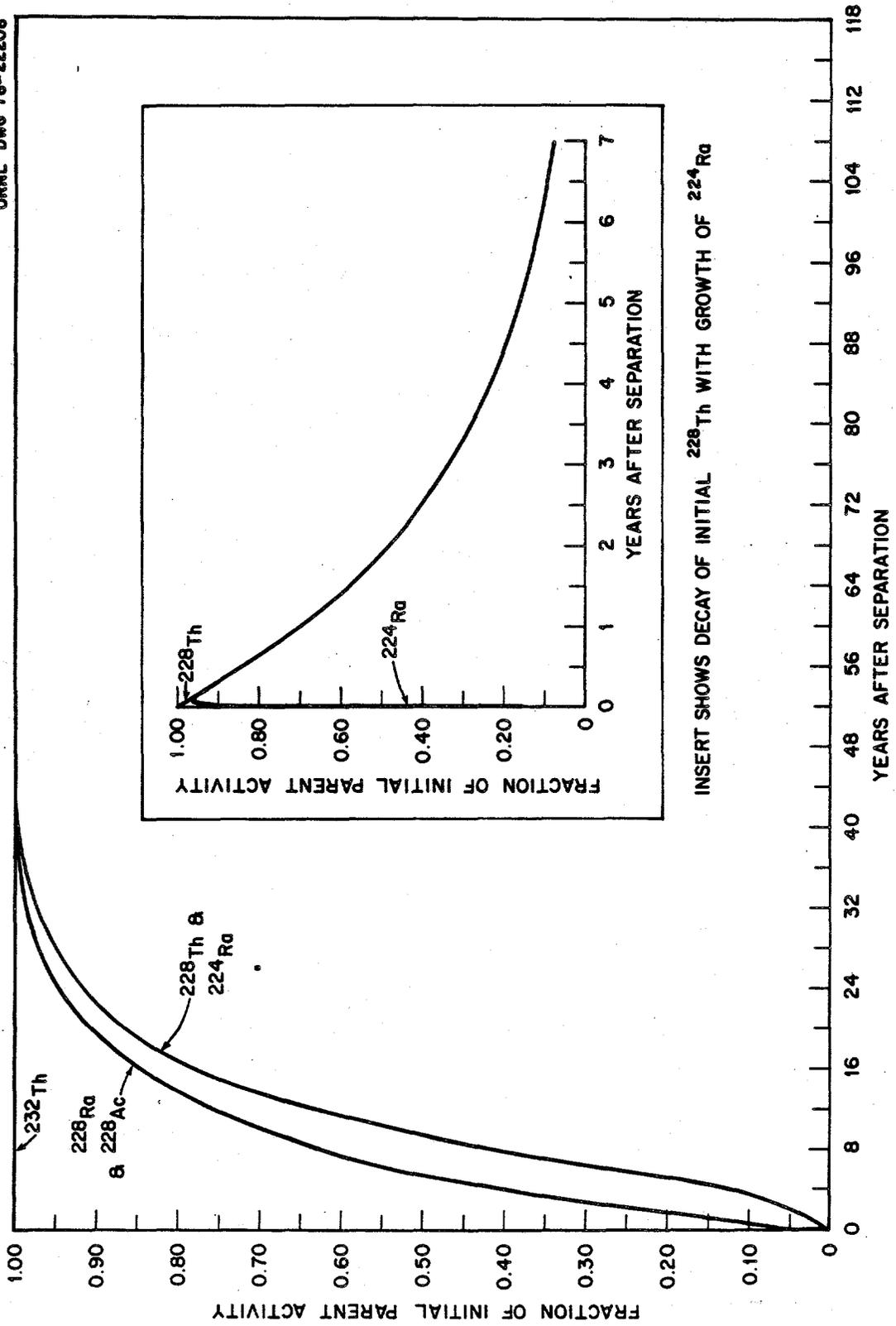


Fig. 1. Thorium-232 series.

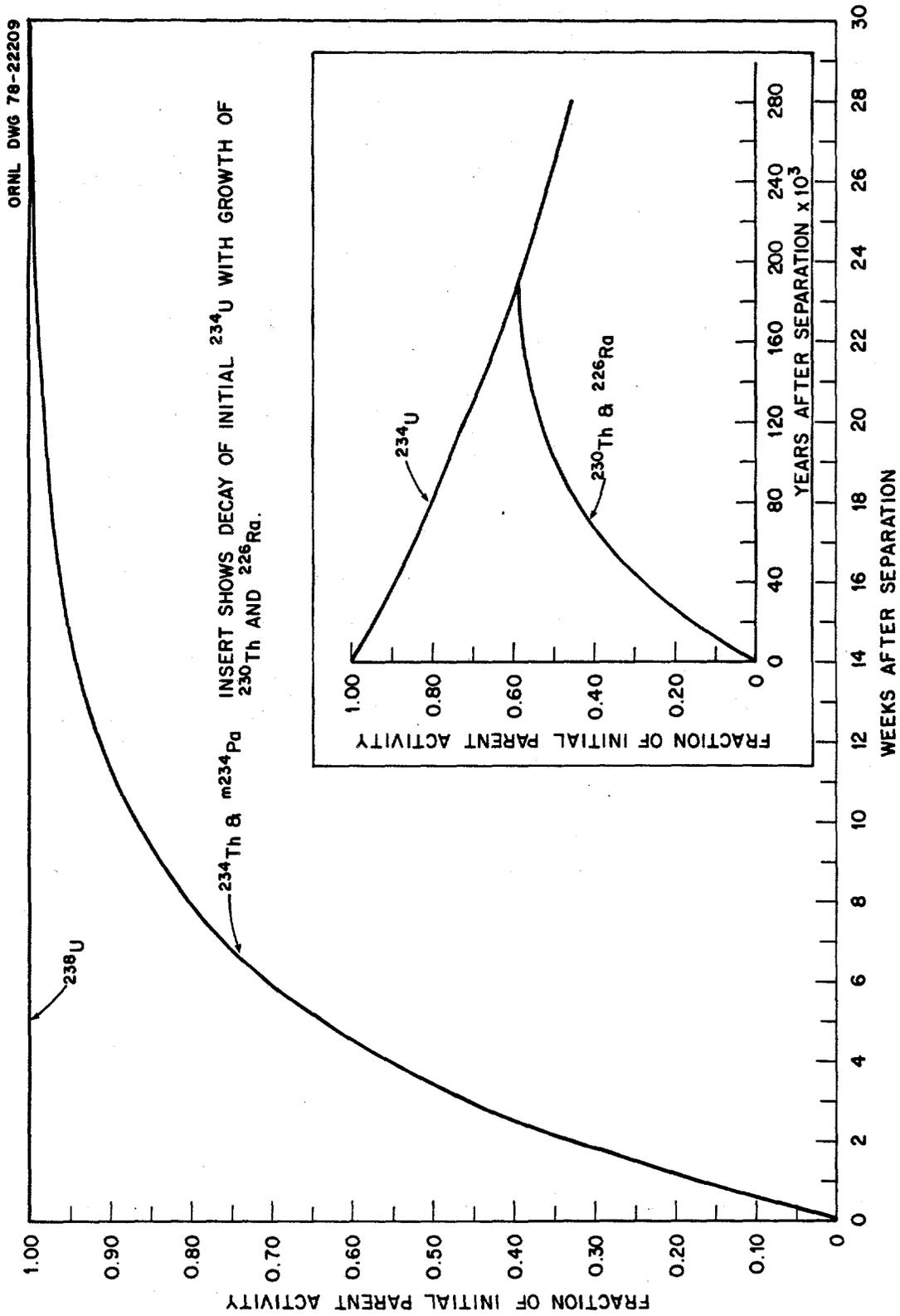


Fig. 2. Uranium-238 series.

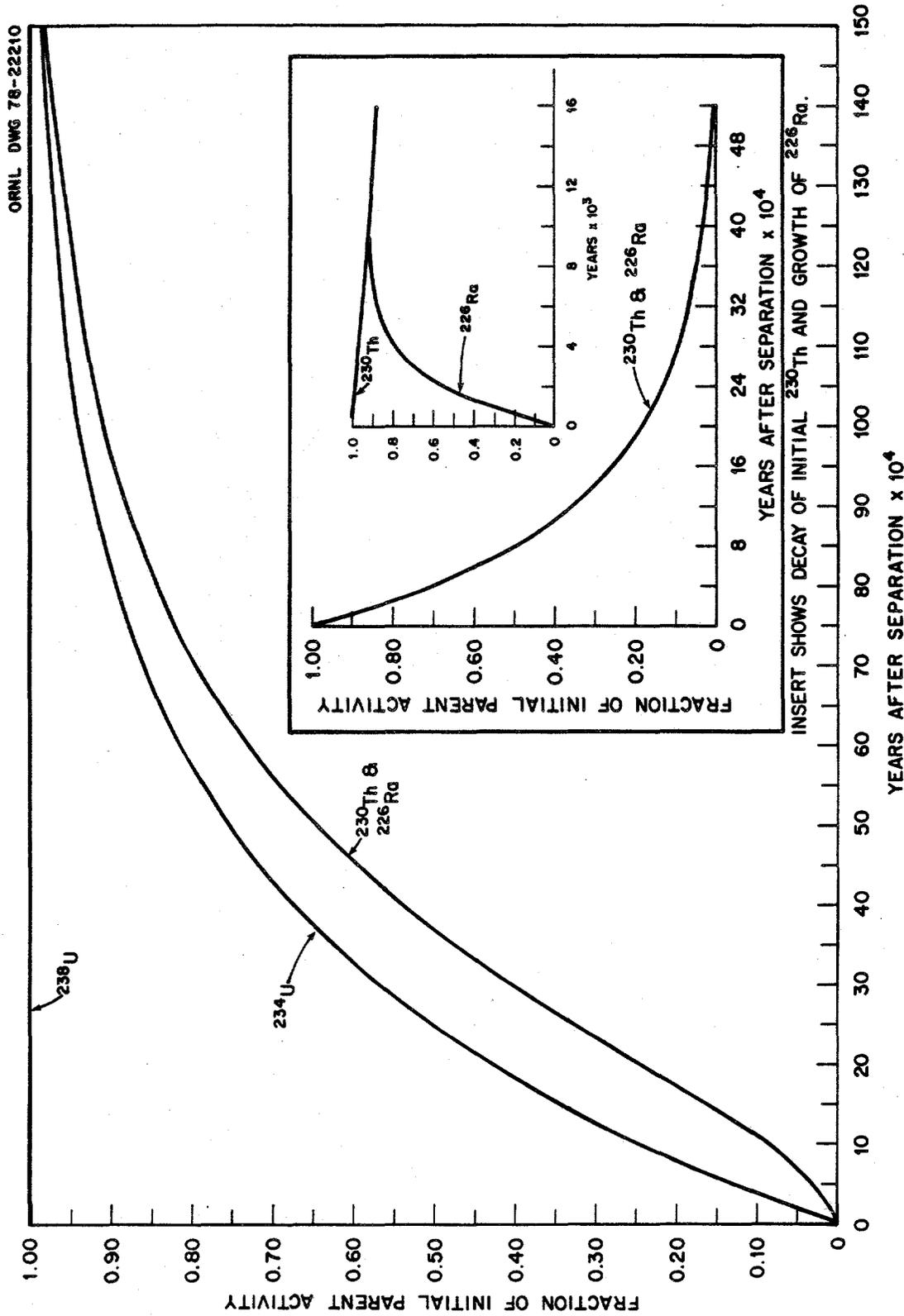


Fig. 3. Uranium-238 series.

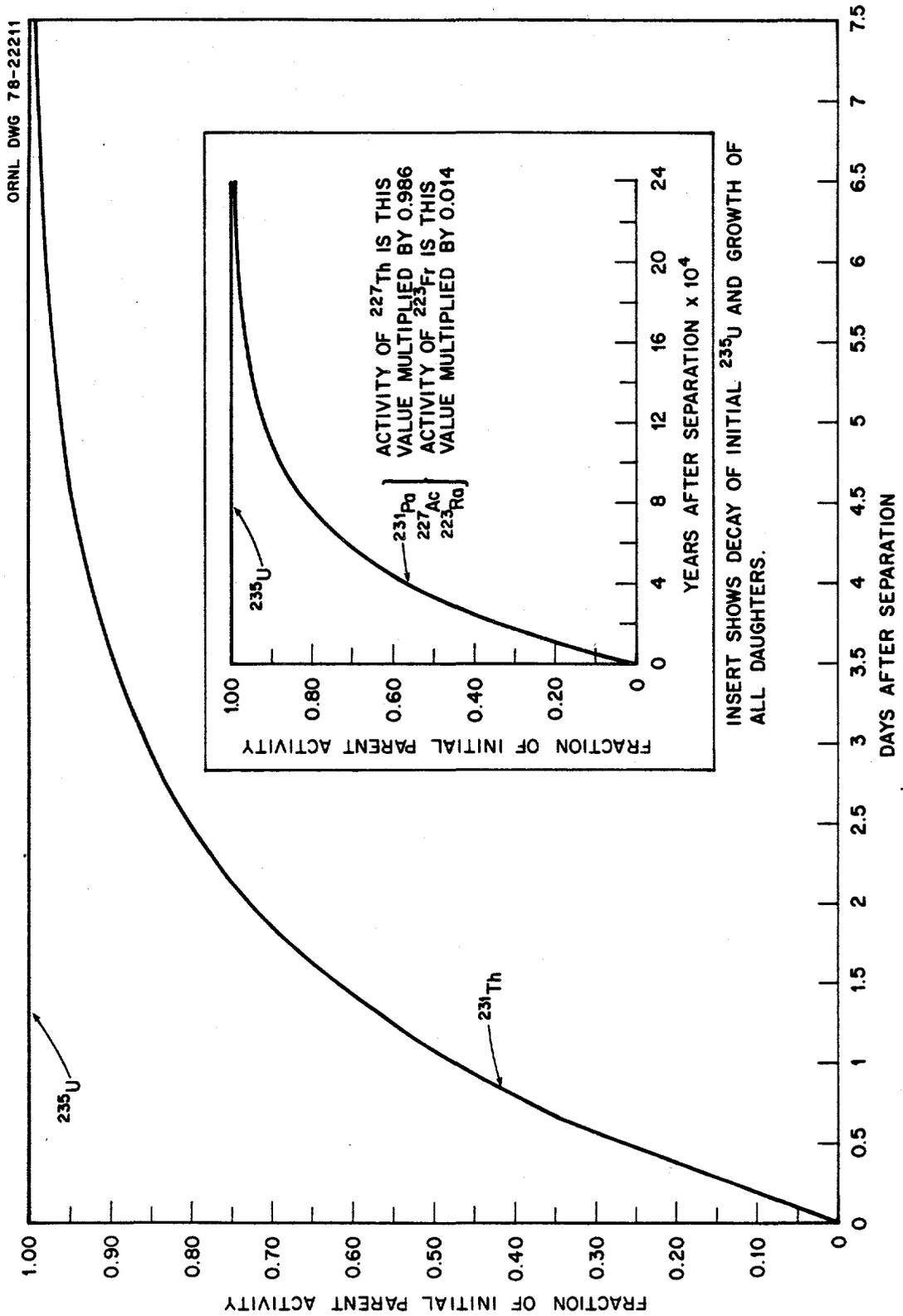


Fig. 4. Uranium-235 series.

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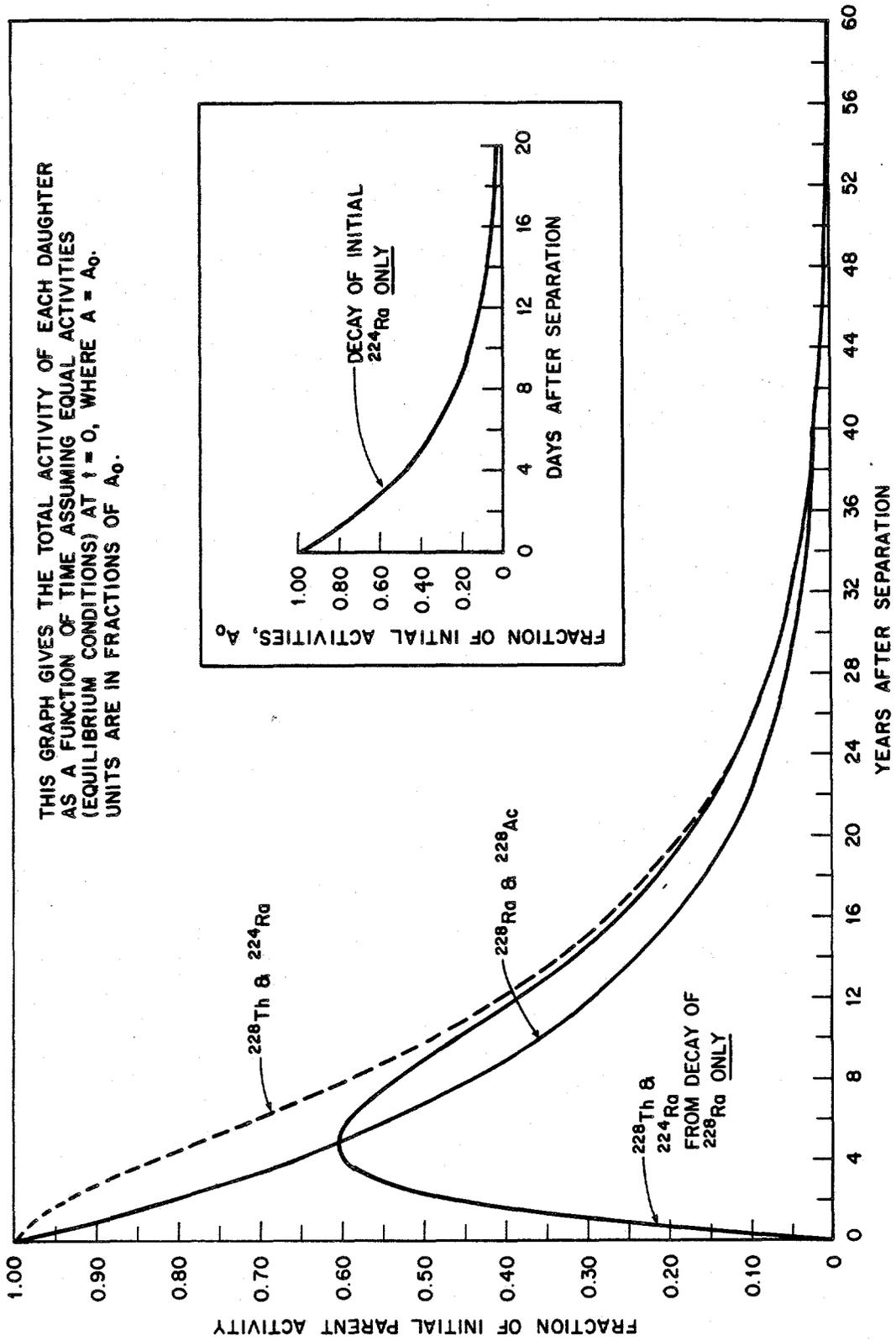


Fig. 5. Decay of thorium-232 series in tailings material.

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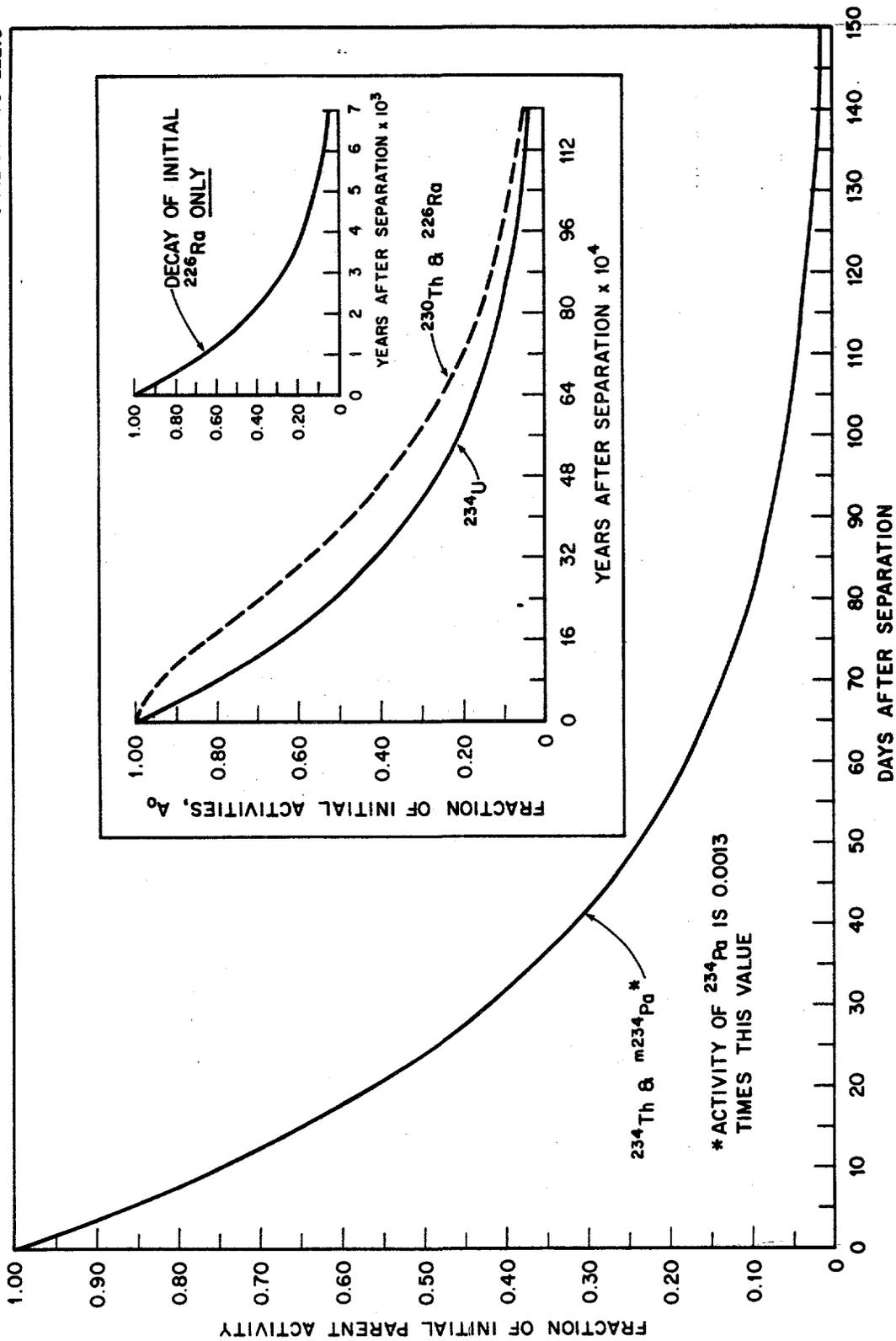


Fig. 6. Decay of uranium-238 decay series in tailings material.

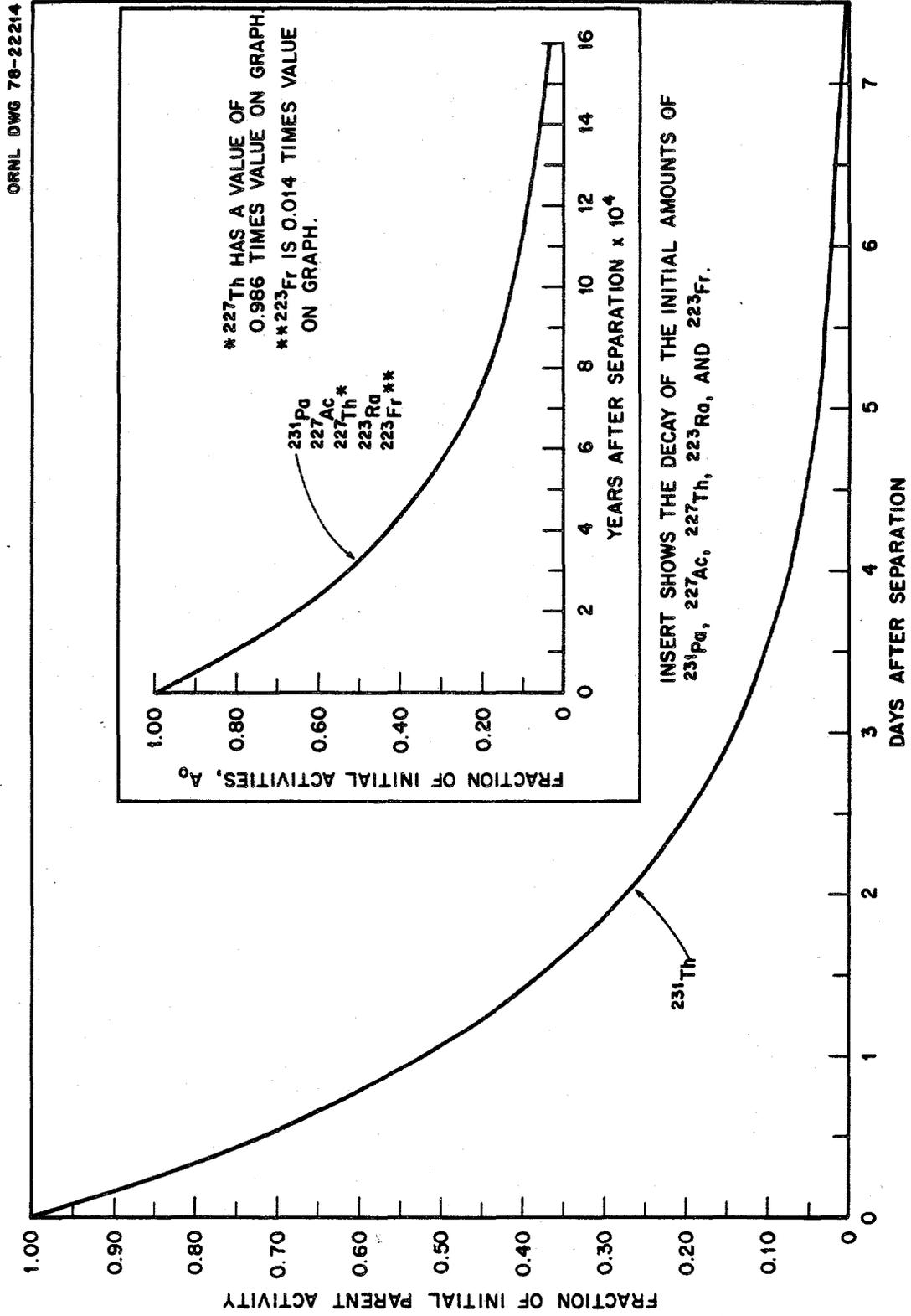


Fig. 7. Decay of uranium-235 series in tailings material.

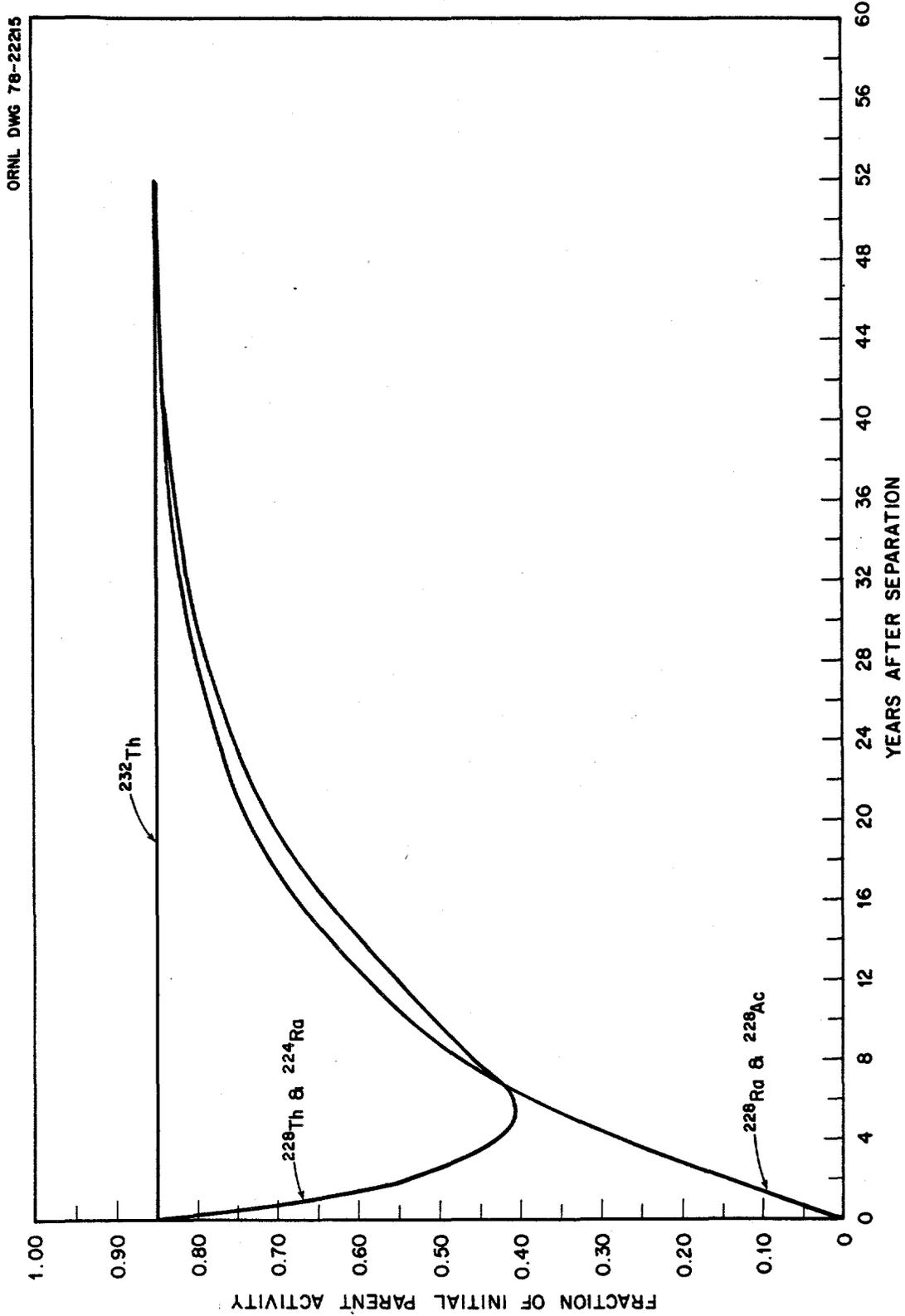


Fig. 8. Thorium-232 decay series. 85% separation displays concentration in finished product.

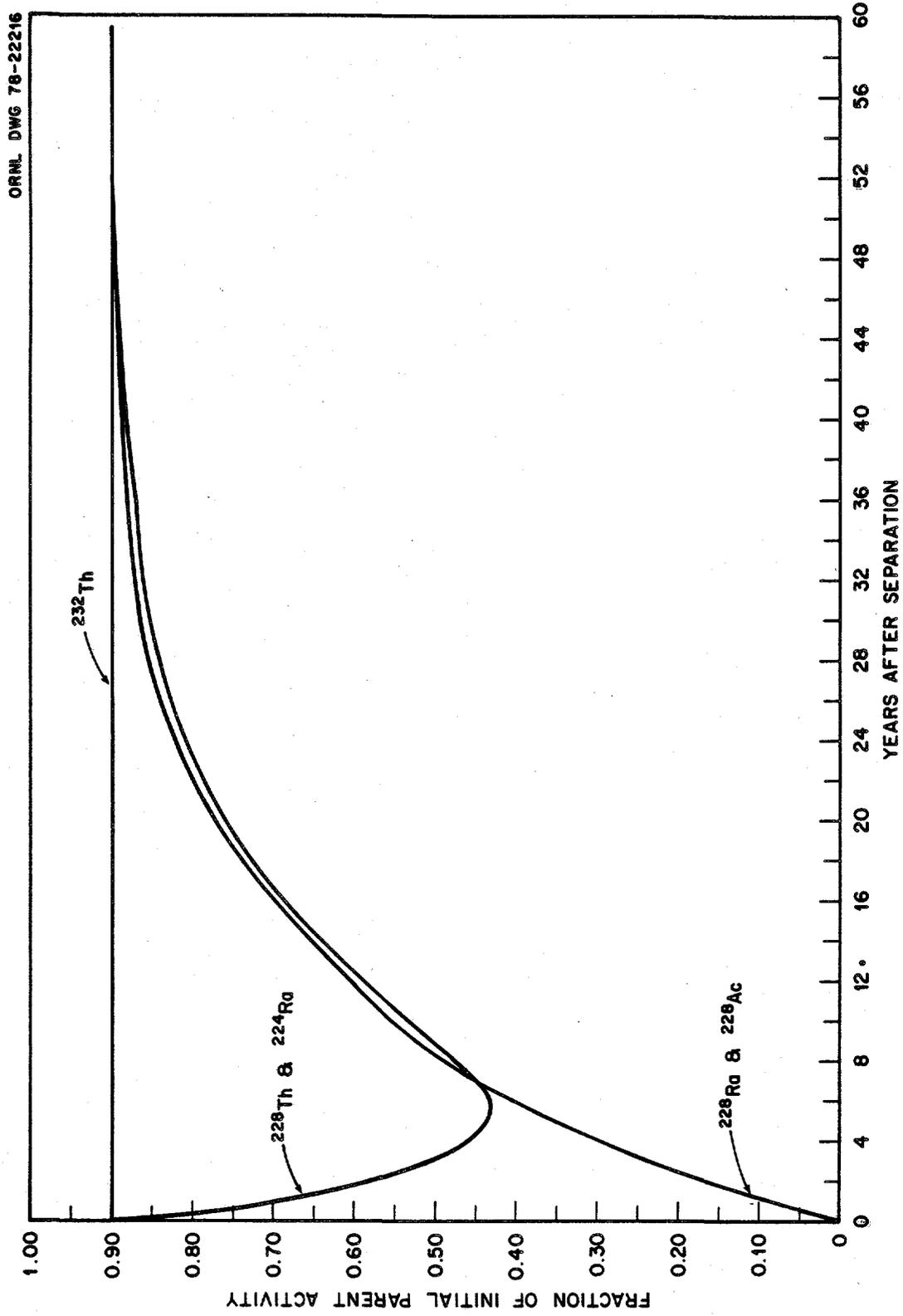


Fig. 9. Thorium-232 decay series. 90% separation. Displays concentration in finished product.

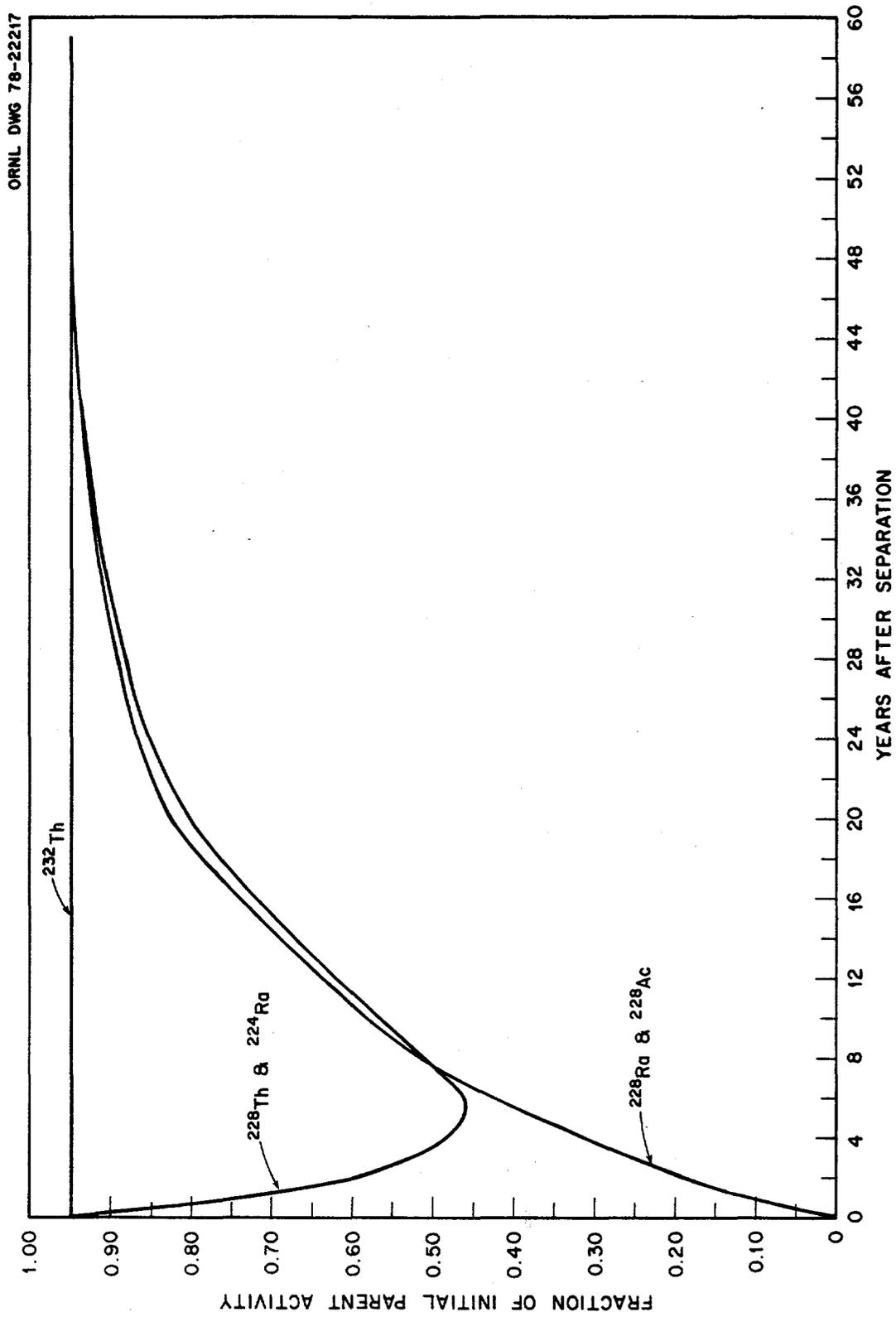


Fig. 10. Thorium-232 decay series. 95% separation. Displays concentration in finished product.

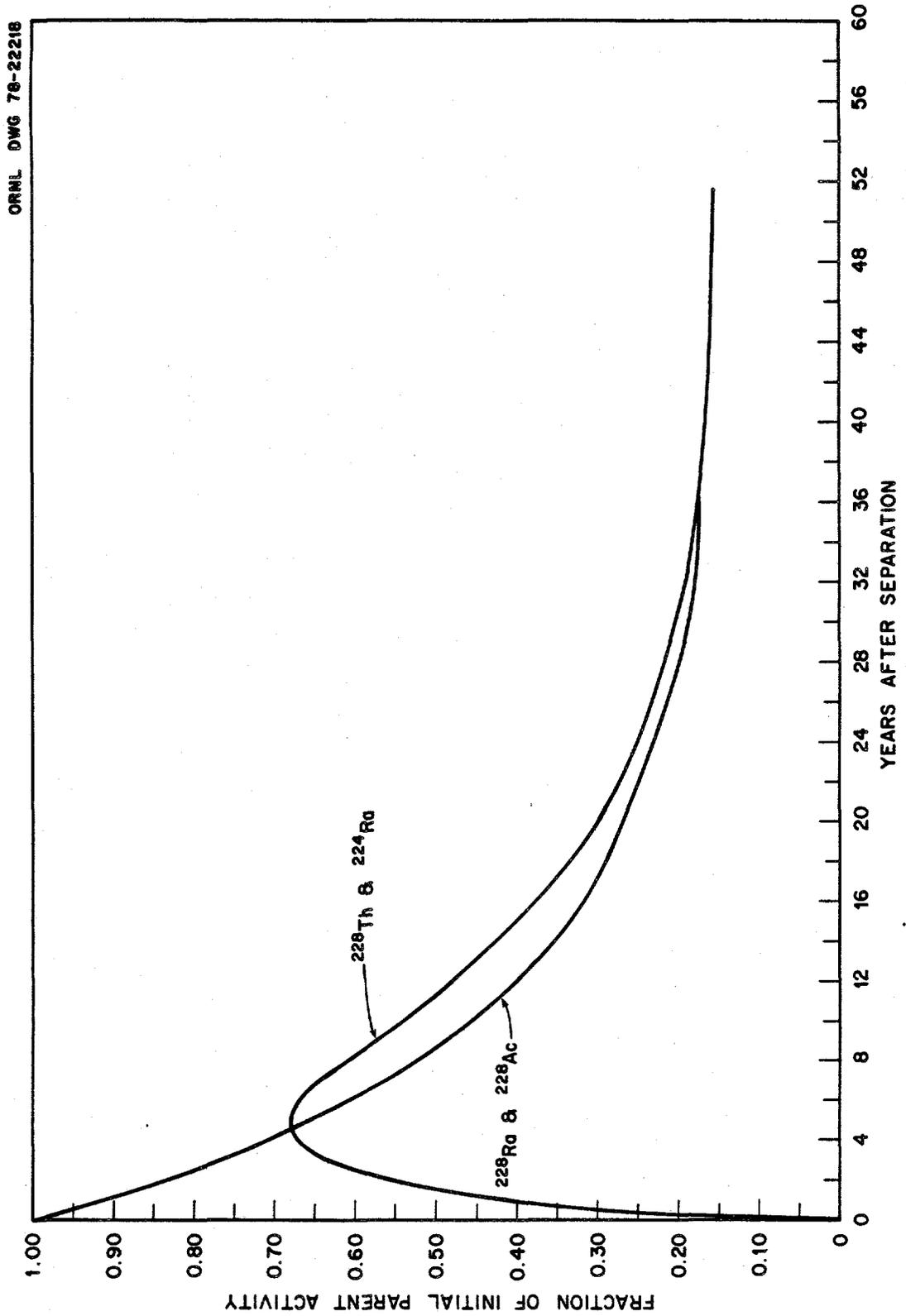


Fig. 11. Thorium-232 decay series. 85% separation. Displays concentration in tailings material ( $^{232}\text{Th}$  concentration constant at 0.15).

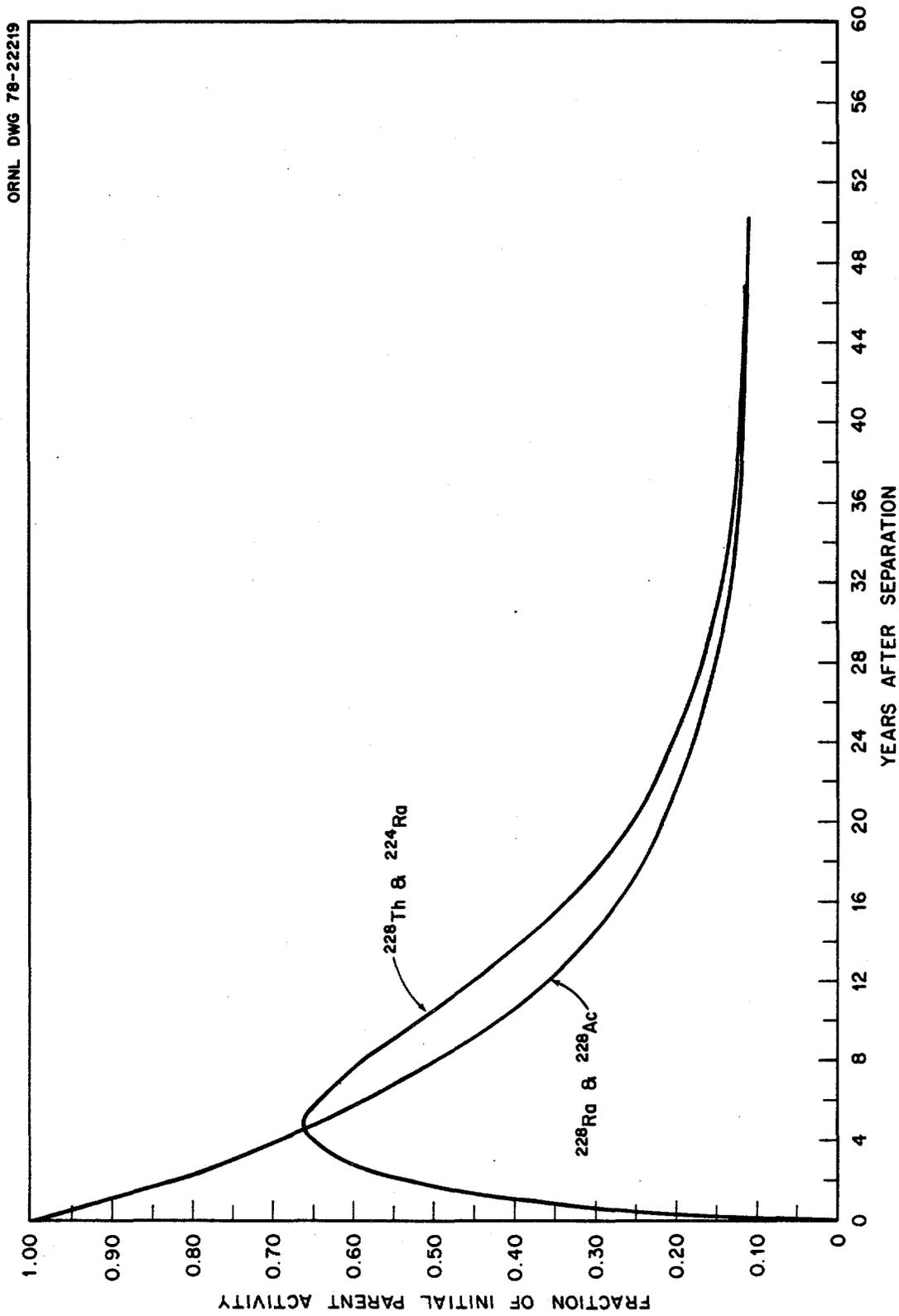


Fig. 12. Thorium-232 decay series. 90% separation. Displays concentration in tailings material. ( $^{232}\text{Th}$  concentration constant at 0.10).

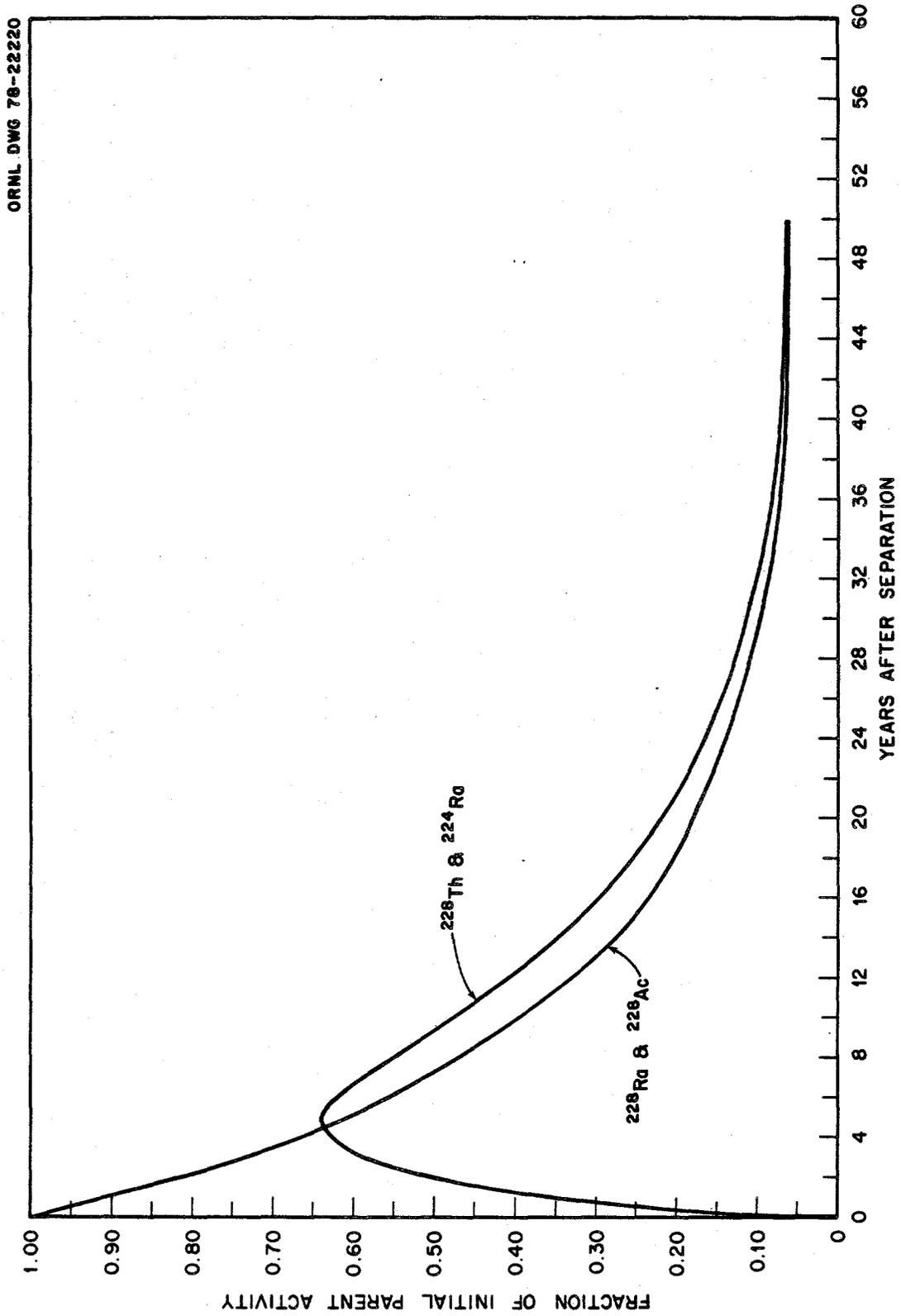


Fig. 13. Thorium-232 decay series. 95% separation. Displays concentration in tailings material. ( $^{232}\text{Th}$  concentration constant at 0.05).

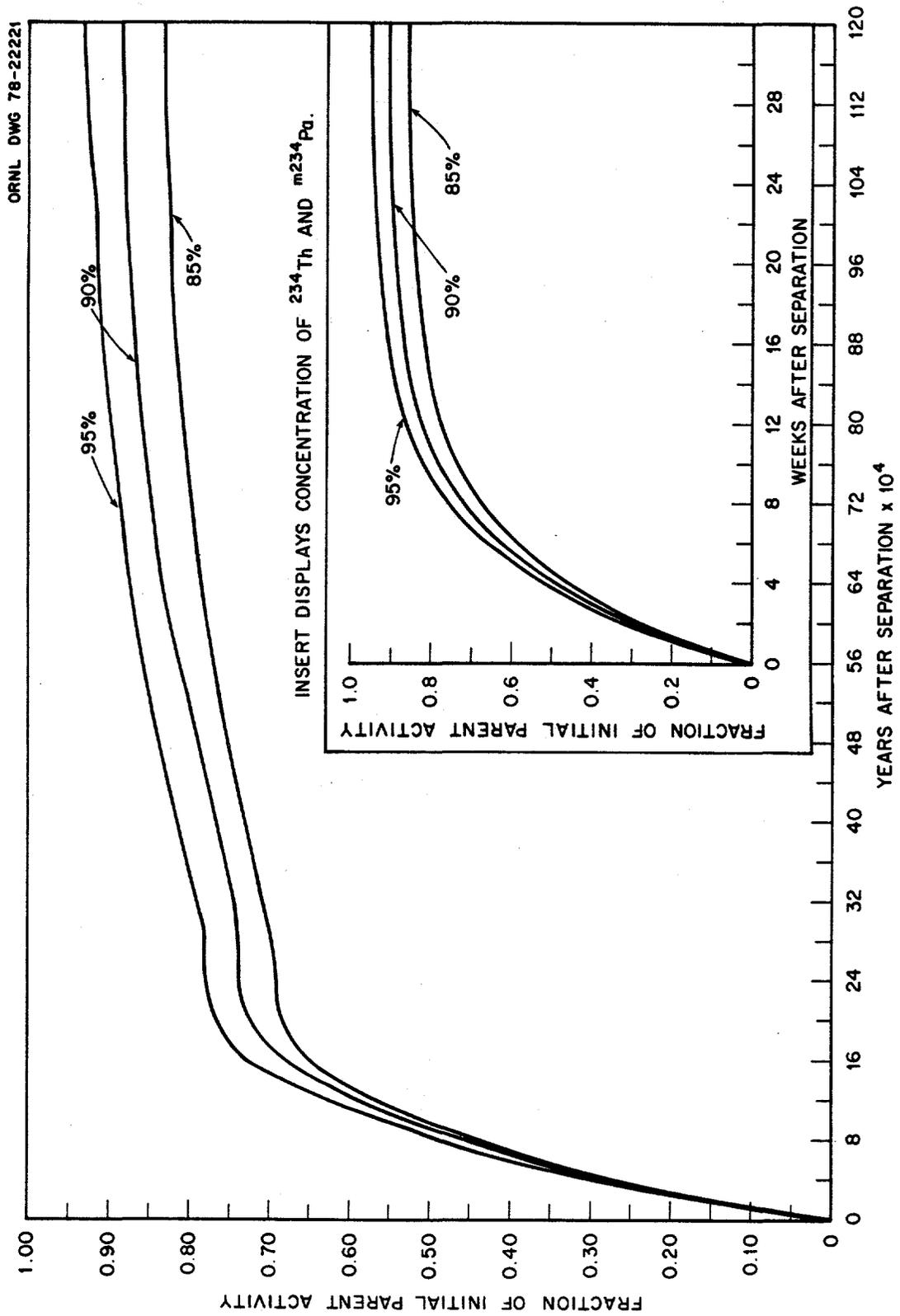


Fig. 14. Uranium-238 decay series. Displays concentration of  $^{230}\text{Th}$  and  $^{226}\text{Ra}$  in finished product.

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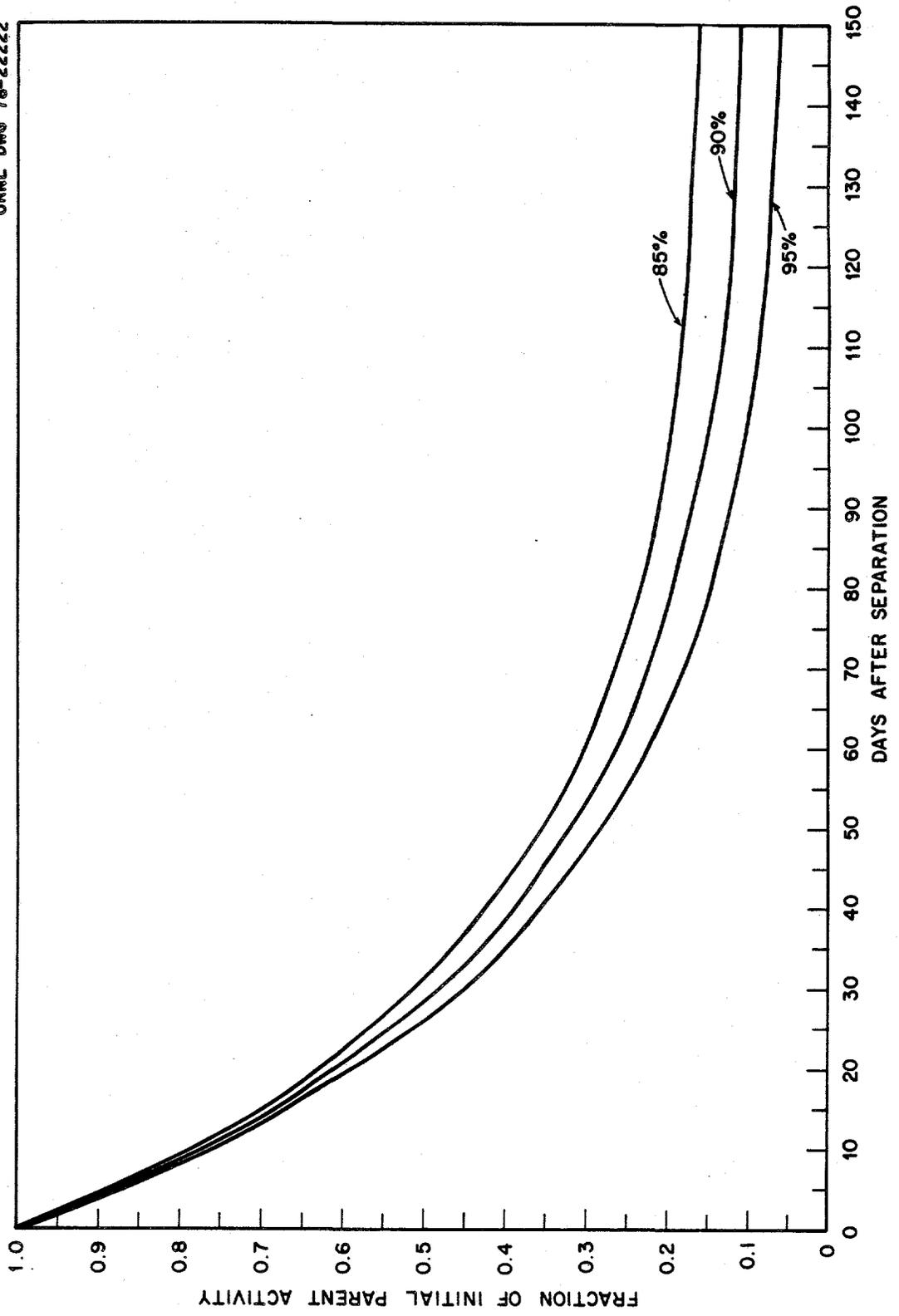


Fig. 15. Uranium-238 decay series. Displays concentration of  $^{234}\text{Th}$  and  $^{234\text{m}}\text{Pa}$  in tailings material.

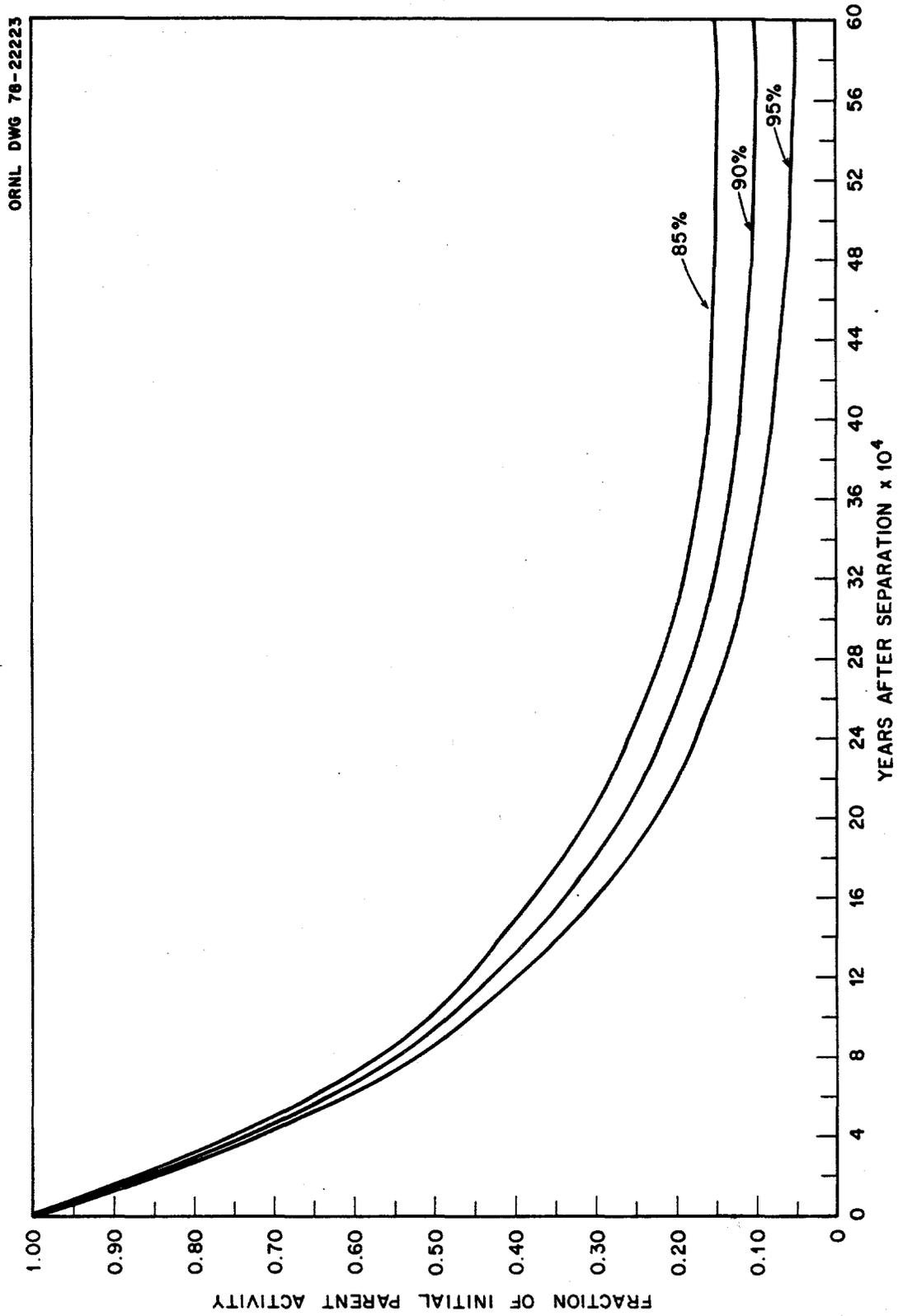


Fig. 16. Uranium-238 decay series. Displays concentration of  $^{230}\text{Th}$  and  $^{226}\text{Ra}$  in the tailings material.

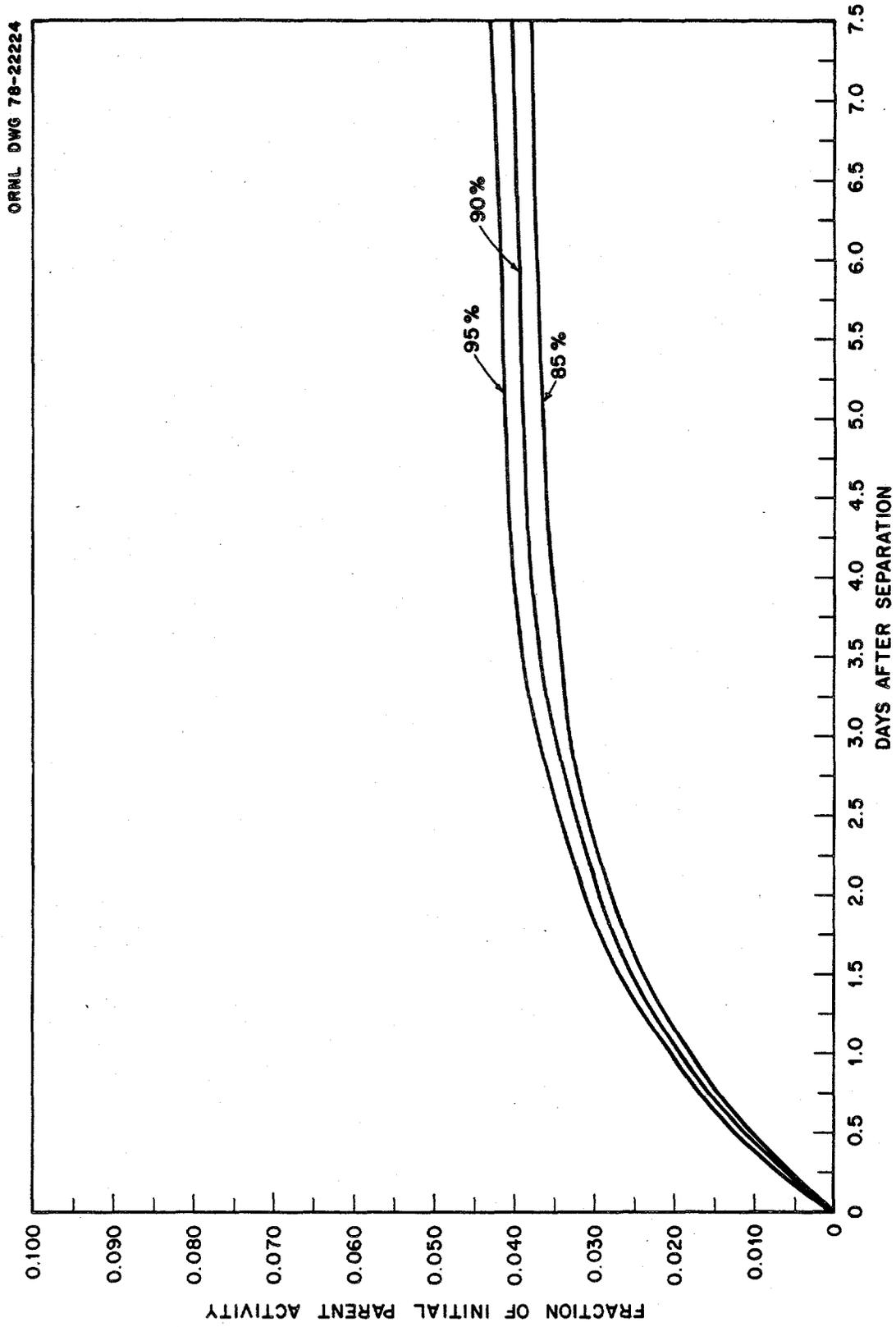


Fig. 17. Uranium-235 decay series. Displays concentration of  $^{231}\text{Th}$  in the finished product.

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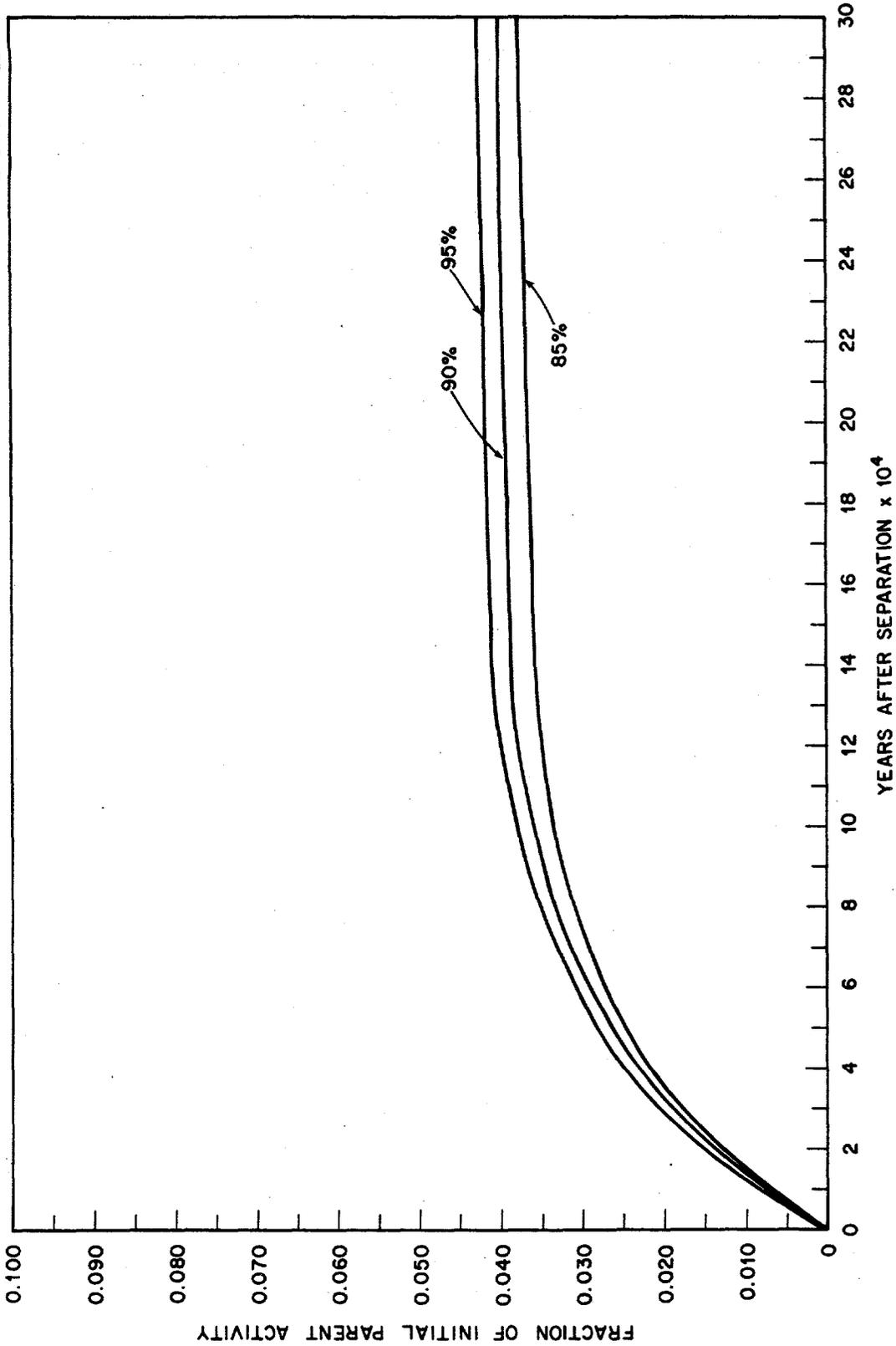


Fig. 18. Uranium-235 decay series. Displays concentration of <sup>231</sup>Pa, <sup>227</sup>Ac, and <sup>223</sup>Ra in the finished product. (Value for <sup>237</sup>Th is 0.986 times graph value.)

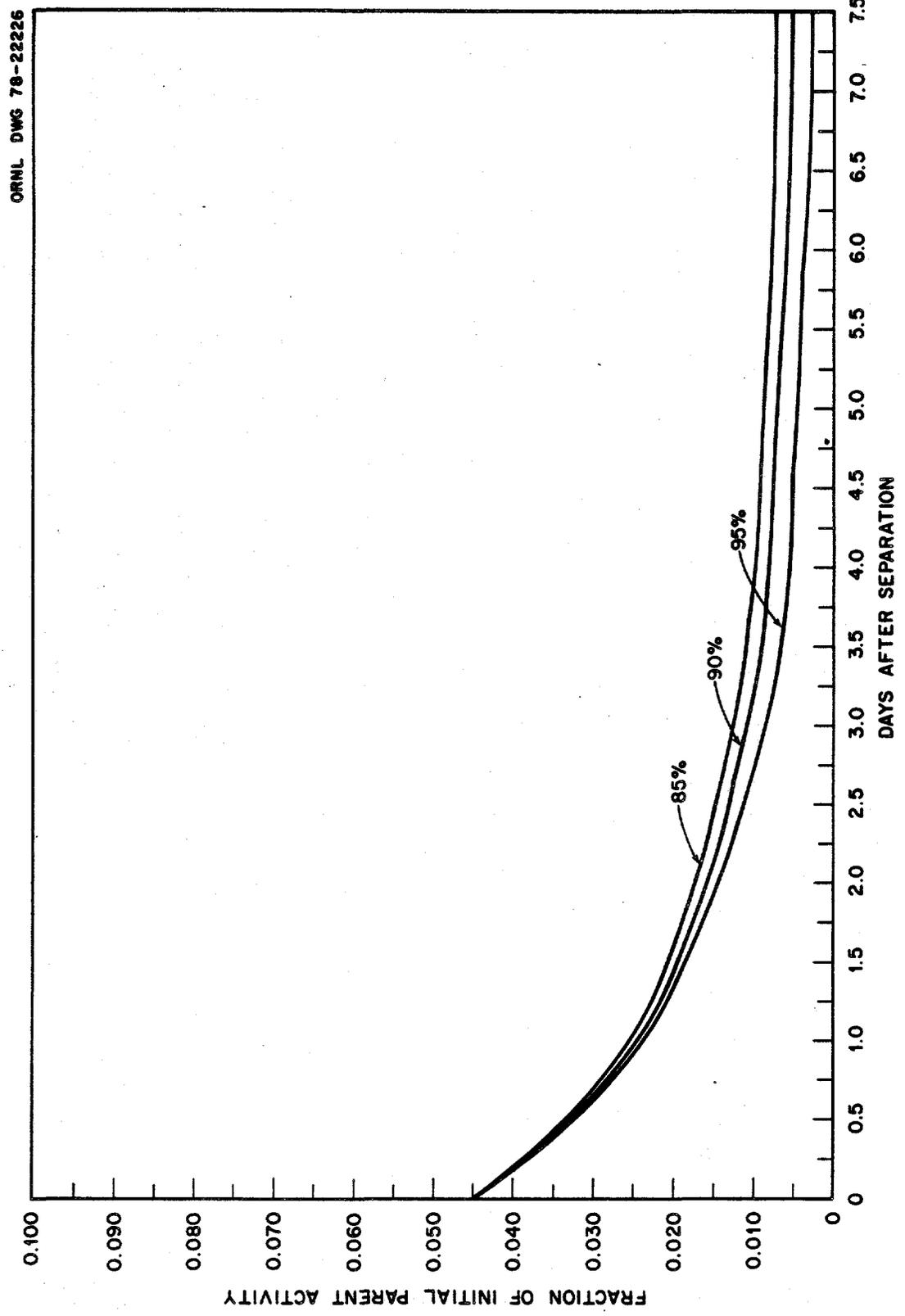


Fig. 19. Uranium-235 decay series. Displays concentration of  $^{231}\text{Th}$  in the tailings material.

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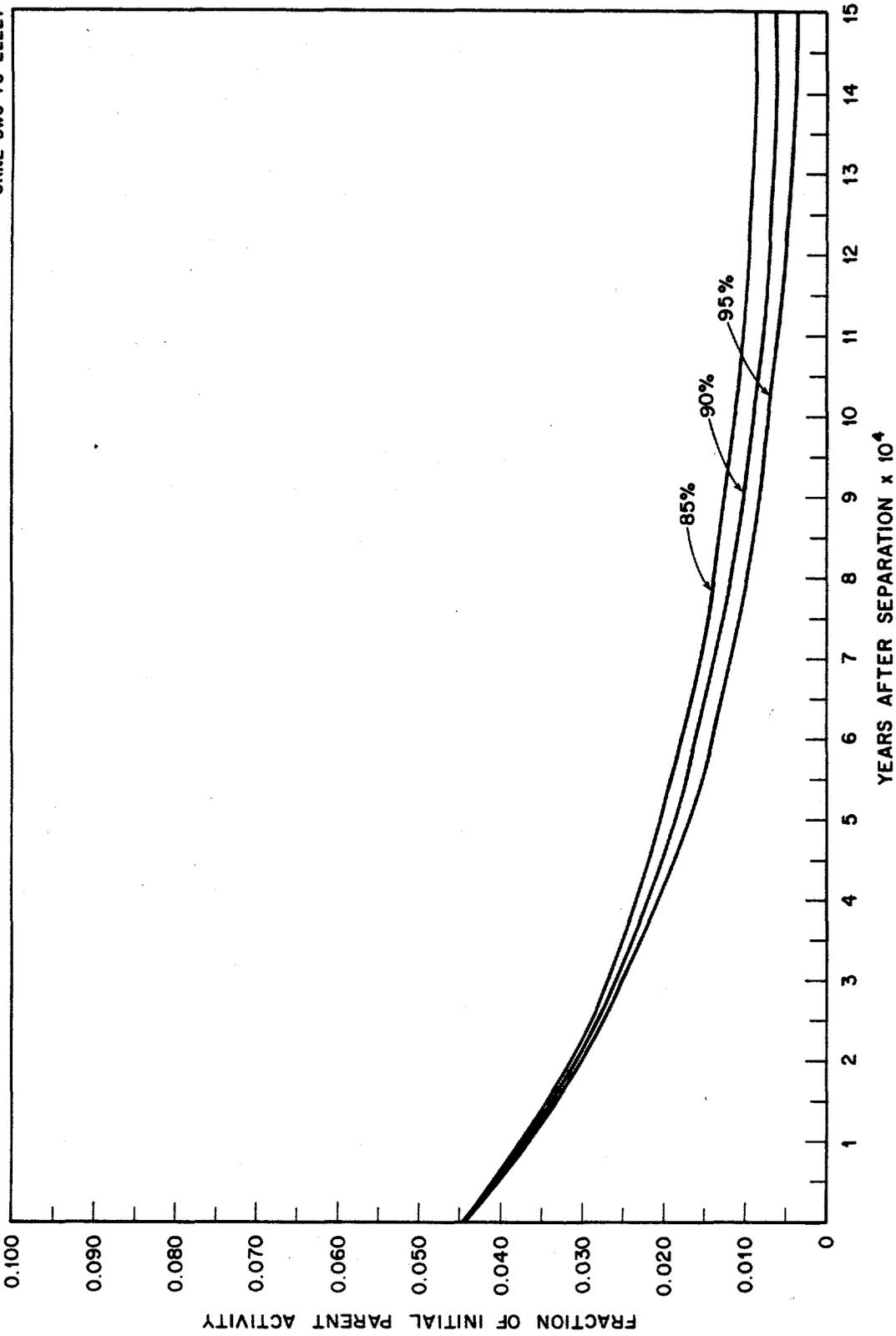


Fig. 20. Uranium-235 decay series. Displays concentration of  $^{231}\text{Pa}$ ,  $^{227}\text{Ac}$ , and  $^{223}\text{Ra}$  in tailings material. (Value for  $^{227}\text{Th}$  is 0.986 times graph value.)

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