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SUBJECT: THE INTERPRETATION OF WATER INJECTION TESTS IN SITE EVALUATION FOR WASTE DISPOSAL BY HYDRAULIC FRACTURING

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FROM: W. C. McClain

ABSTRACT

A considerable amount of information on the underground behavior of hydraulically induced fractures is contained in the pressure history of the injection phase of the operation. The analysis presented in this report provides the means of interpreting the pressure history of water-injection tests. Such tests can be used therefore to advantage as a part of a program to evaluate the suitability of any proposed site for radioactive waste disposal by hydraulic fracturing.

The analysis includes the theoretical and empirical evaluation of the various components' contribution to the pressure measured in the injection well, including turbulence at the entrance to the fracture, laminar flow viscous losses for circular and noncircular fractures, and the effects of vertical gradients in the confining rock stresses due to tectonism and in the fracture resulting from changes in elevation. These components can be synthesized in various combinations to provide a predicted pressure history or pumping characteristic for almost any possible fracture configuration.

NOMENCLATURE

A, B, S, K, m, n = constants.

c = radius of fracture (ft)

E = modulus of elasticity of rocks (use 10^6 lb/in.^{2}).

f = fanning friction factor (dimensionless).

g = acceleration of gravity (32.2 ft/sec^2).

H = depth below ground surface (ft).

h_e = losses in zone of turbulent flow (lb/in.^{2}).

h_f = losses in zone of laminar flow (lb/in.^{2}).

N_R = Reynolds' number (dimensionless).

P_a = average pressure in fracture (lb/in.^{2}).

P_o = minimum fracture extension pressure (lb/in.^{2}).

P_w = static pressure in injection well at level of injection (lb/in.^{2}).

Q = flow rate (ft^3/sec).

R_H = hydraulic radius (ft).

r = radial coordinate (ft).

r' = point of transition from turbulent to laminar flow.

V = bulk velocity of fluid (ft/sec).

W_{max} = maximum (center) fracture thickness (ft).

δZ = change in elevation (ft).

α = average fracture thickness (ft).

γ = density factor for water (0.433 lb/in.^{2}/ft).

γ_R = density factor for rock.

Δ = cumulative volume injected (ft^3).

θ = dip angle of nonhorizontal fractures.

λ = specific surface energy of rock (use 0.01 ft-lb/in.^{2}).

μ = absolute viscosity of fluid (for water, 2.0886 x 10^{-5} lb-sec/ft^2).

ν = Poisson's ratio for rock (use 0.15).

ρ = bulk density of fluid (for water, 1.95 lb-sec^2/ft^4).

$\sigma_z, \sigma_x, \sigma_y$ = vertical and horizontal components of original earth stress.

ψ = narrow, restricted dimension of noncircular fracture (ft).

I. INTRODUCTION

Hydraulic fracturing can now be considered an acceptable, indeed a preferred, method for the disposal of intermediate-level radioactive wastes at the Oak Ridge National Laboratory. In testimony to this is the fact that since December 1966 the Operations Division has disposed of 328,000 gal of waste, containing a total of 120,000 curies of activity, using the hydraulic fracturing facility in Melton Valley on a routine basis. These operations marked the culmination of an extended developmental program[1,2] carried out by the Health Physics Division which succeeded in solving many technical problems[3,4] and demonstrated the value of the technique as a safe, permanent, and economical waste disposal method.

However, it is not possible to guarantee the applicability of this technique at any other location, even in an adjacent valley, without first carrying out a site testing and evaluation program. This is because the technique is predicated upon the formation of essentially horizontal fractures deep in the ground when the waste slurries are pumped through a slot in the cased injection well. The development of vertical fractures would represent a breach of the containment and isolation barrier provided by the overlying rocks, thereby negating the principal advantage of hydraulic fracturing. The orientation of hydraulically induced fractures is controlled primarily by the state of stress in the earth's crust;[5] and, since there is no way to predict that stress state, it will be necessary to prove that the induced fractures are horizontal at any proposed waste-disposal site as a part of the site examination procedure.

At the Oak Ridge facility, the horizontality of the fractures was confirmed by the core drilling which was conducted as part of the experimental program. In this case, several grout injections, each identified by a chemical dye and an isotopic tag, were intersected at approximately the same depth in a number of core holes drilled from the surface. At any other proposed hydraulic fracturing site, it may be necessary to perform a similar core drilling program in order to conclusively prove that the fractures are horizontal. This core drilling program is both time consuming and very expensive. The purpose of this investigation was, therefore, to develop a method whereby a fairly reliable indication of the fracture orientation could be obtained by some other method. In this way, it should be possible to significantly reduce the number of core holes and, hence, the expense of proving the applicability of hydraulic fracturing at a proposed site.

The principal data obtained from a hydraulic fracturing operation are: (1) the fluid pressure in the well required to initially fracture the rock formation (breakdown pressure); (2) the pressure, flow-rate, and volume history of the injection phase; and (3) the pressure-time function when the well is shut-in following completion of the injection. Traditionally, the breakdown pressure and the shut-in pressure, especially the so-called "instantaneous" shut-in pressure, have been used to evaluate the fracture orientation, based on the premise that if either of these pressures is less than the overburden pressure, the fracture cannot be horizontal. If both of these pressures are greater than the overburden pressure, the situation is ambiguous; for, in this case, the fracture may be either horizontal or vertical. In practice, it has been found to be extremely difficult to obtain reliable measurements for either the breakdown or the shut-in pressure. In the case of the breakdown pressures, this may be due to the complex stress field around the borehole, accentuated influences from the imperfections in the surface of the hole or the fit of the packers, variations in the tensile strength of the rock, or other similar factors. Interpretation of the shut-in pressure data is similarly clouded by the effect of fluid leak-off into the rock formation. Because of these difficulties, the interpretation of breakdown and shut-in pressures alone is usually not adequate to yield a definition of the fracture orientation for waste disposal operations. Therefore, this investigation was directed mainly toward the interpretation of the third body of data--the pressure history of the injection phase.

For several reasons, it appears advantageous to obtain the necessary data from a preliminary water-injection test similar to the water tests conducted at the Oak Ridge facility (for a different purpose). These water injections can be carried out more easily and at a much lower cost than equivalent slurry injections. Furthermore, the non-Newtonian fluid properties of cement-based slurries[6] make the interpretation of grout-injection data extremely complex, if not impossible, whereas water-injection data can be subjected to a fairly detailed dynamic analysis.

Although the material in this report is concerned only with the interpretation of water-injection-test data, it must be realized that these tests comprise only a part of a complete site testing program. Before a water test is carried out, a considerable amount of work must have been completed. This work includes the selection of a favorable site, based on consideration of the geology of the area, and selection of host formation(s) at depth, based on analysis of data from at least one core hole. If the water-injection data suggests that horizontal fractures are the normal mode for the site, it will probably be necessary to confirm this fact by injecting a tagged slurry and then intersecting the grout sheet with at least a few core holes from the surface. The advantages gained from the water tests are a much increased level of confidence, thereby reducing the risk associated with the investment in the confirmatory testing, and a significant reduction in the number of core holes required to provide conclusive evidence of the horizontality of the fractures.

The following sections discuss a number of possible fracture configurations. This discussion is largely theoretical and therefore based on simplifying assumptions which are necessary to render the situation amenable to analysis. In most cases, the validity of the assumption and the effect of possible deviations in the prototype are discussed as they arise. Two of these simplifying assumptions form the basis of the entire approach. These are that the rocks into which the injections are made behave as a homogeneous elastic solid and that they have zero permeability. Although the shales at the Oak Ridge site are certainly not ideally elastic, the deformations and stresses encountered in this treatment are very small (that is, of the order of 0.05 in. and 10 psi, respectively); and therefore the assumption of elasticity should be valid as a first approximation. Detailed examination of the shale indicates that its permeability is measurable but very small (less than 0.005

millidarcies[7]). Therefore, the leak off of fluid into the formation should have a negligible effect on the pressures during the injection phase.

II. HORIZONTAL CIRCULAR FRACTURES

The simplest configuration of the fracture for waste-disposal operations is a single horizontal fracture extending outward equally in all directions from the injection well; that is, a circular fracture, which is parallel to the surface and axially symmetrical about the injection well. There is some evidence[7] to suggest that this is also the configuration actually obtained, at least approximately, at the Oak Ridge facility. The pressure relationships in this type of fracture are illustrated on Fig. 1, where P_w is the fluid pressure at the center of the injection well at the level of the slot, h_e is the pressure loss in the turbulent region resulting from both entrance and friction losses, and h_f is the friction loss in the remainder of the fracture where the flow is laminar. Since the pressure at the tip of the fracture ($r = c$) must be almost exactly equal to the overburden pressure (σ), the fluid pressures at the various locations indicated on Fig. 1 can be expressed

$$P_3 = \sigma$$

$$P_2 + \frac{\gamma V_2^2}{2\,g} = P_3 + h_f + \gamma\,\delta Z \qquad (1)$$

$$P_w = P_2 + \frac{\gamma V_2^2}{2\,g} + h_e + \gamma\,\delta Z$$

where

γ = density factor for water (0.433 psi/ft).

Since the fracture is assumed to be horizontal, there is no difference in elevation between the center of the well and the tip of the fracture and the $\gamma\,\delta Z$ terms of Eqs. (1) reduce to zero. The radial velocity at the center of the injection well ($r = 0$) is, from symmetry, equal to zero. The analysis will be concerned primarily with those cases where the fracture has extended some distance from the injection well; that is, where the velocity of the fracture tip is greatly reduced

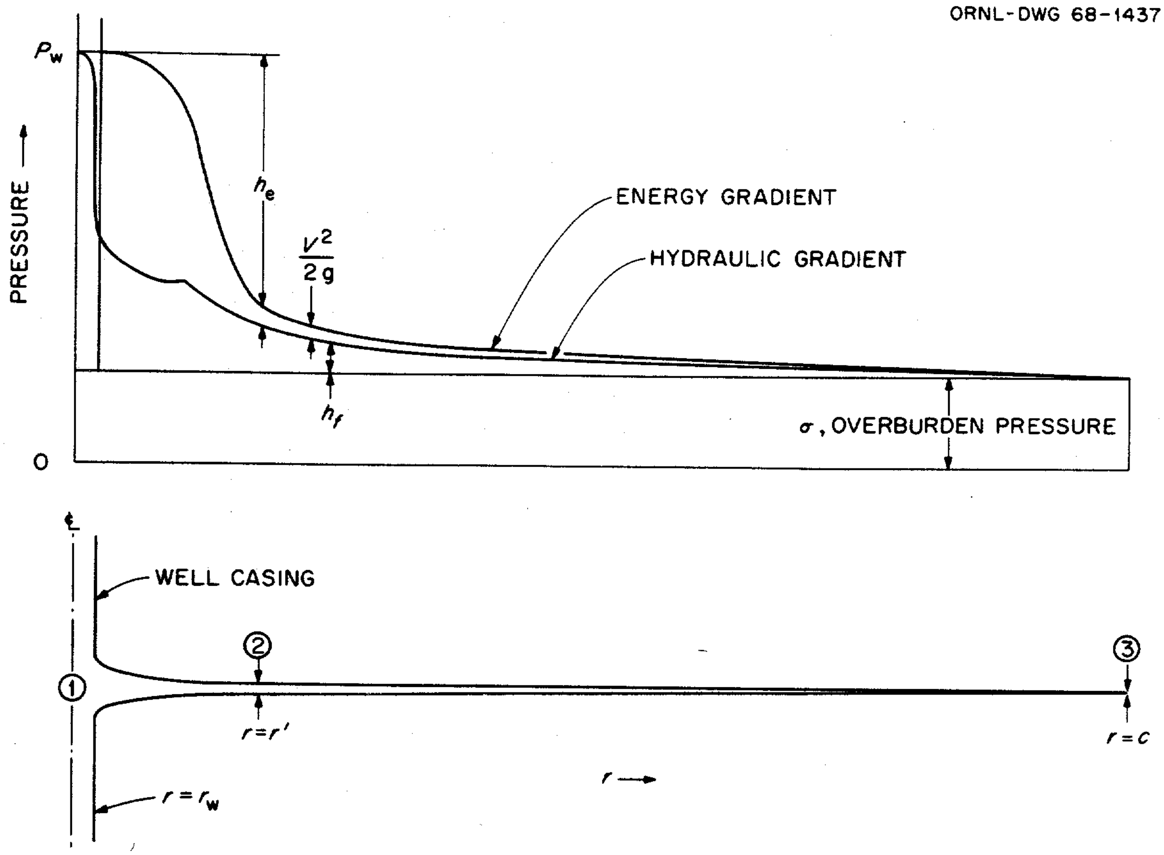

Fig. 1. Pressure Relationships for a Circular Horizontal Fracture.

and its velocity head is approximately equal to zero. Equation (1) therefore reduced to

$$P_w = \sigma + h_e + h_f \ . \tag{2}$$

The analysis will be developed from this point by examining the flow conditions in the fracture at a given instant of time as though the fluid were flowing in the fracture as it would in a rather oddly shaped conduit. However, the fluid is actually creating the fracture (the conduit) as it moves along, and the situation is not strictly analogous to the usual steady-state fluid dynamics case. The analysis based on this quasi-dynamic situation should closely approximate the true state of affairs (after adding an increment for the energy absorbed in actually fracturing the rock--see page 22) since the true shape of the fracture (ellipsoidal) is incorporated in the derivations.

Since the overburden pressure (σ) in Eq. (2) can be estimated satisfactorily from the depth, it remains only to evaluate the two components of the losses h_e and h_f. The zone of turbulent flow can be defined as the region where Reynold's number (N_R) is greater than 2×10^3. Since the hydraulic radius (R_H) for flow between parallel plates[8] is

$$R_H = \frac{\alpha}{2} \tag{3}$$

where

α = separation of the plates or, in this case, the average thickness of the fracture,

the Reynold's number will be given by

$$N_R = \frac{4\ R_H\ V\ \rho}{\mu} = \frac{2\ \alpha\ V\ \rho}{\mu} \tag{4}$$

where

ρ = bulk density in $\text{lb-sec}^2/\text{ft}^4$ and

μ = absolute viscosity of fluid in lb-sec/ft^2.

From the geometry of axially symmetrical radial flow, the bulk velocity as a function of radial distance (r) will be

$$V = \frac{Q}{2\pi\alpha r} \tag{5}$$

where

Q = flow rate (ft^3/sec). Combining Eqs. (4) and (5), the expression

$$N_R = \frac{Q\rho}{\pi r \mu} \quad (6)$$

is obtained, which indicates that for this situation Reynolds' number is not a function of crack thickness (α). The limit of the turbulent zone will therefore be given by

$$r' = \frac{Q\rho}{2000\ \pi\mu} \quad (7)$$

The friction losses (h_f) in the laminar region ($r' \leqq r \leqq c$) can be found by examining the pressure drop (dP) across a "ring" of differential width dr, with the assumption that the fluid is moving between parallel plates of separation α. This pressure differential can be written

$$dP = \frac{\gamma}{2\ g} (V^2_{r+d_r}) - \frac{\gamma}{2\ g} V_r^{\ 2} + \frac{fV^2\rho}{288\ R_H} dr \quad (8)$$

where

f = fanning friction factor.

In order to be strictly correct, Eq. (8) should include a component arising from the viscosity effects of axial flow. This factor results from the energy required to increase the horizontal distance separating two points as they move from one side of the differential element to the other. Since this is a second order viscosity effect, it has been neglected.

Since the friction factor for laminar flow between parallel plates is

$$f = \frac{24}{N_R} = \frac{12\ \mu}{\alpha V \rho} \quad (9)$$

and, in the laminar flow region, the velocity heads are insignificant, Eq. (8) reduces to

$$dP = \frac{\mu Q}{24\ \pi\alpha^3 r} dr\ . \quad (10)$$

The pressure at r = r' will then be the integral of Eq. (10) between r' and the tip of the fracture (c).

$$P_{r'} - \sigma = h_f = \int_{r'}^{c} \frac{\mu Q}{24\ \pi\alpha^3 r}\, dr \tag{11}$$

$$h_f = \frac{\mu Q}{24\ \pi\alpha^3} \ln \frac{c}{r'}$$

Sneddon[9] has shown that a flat, circular crack with internal pressure in an infinite elastic body will assume an elliptical cross section and that the center thickness of this flattened ellipsoid (W_{max}) will be a function of the average pressure (P_a) regardless of the actual pressure distribution in the fracture, according to

$$W_{max} = \frac{8(1 - \nu^2) c\ P_a}{\pi E} \tag{12}$$

where

ν = Poisson's ratio of the rocks and
E = modulus of elasticity (lb/in.^{2}).

The influence of the ground surface is ignored by the assumption of an infinite (rather than semi-infinite) elastic body. This simplification materially increases the generality of later conclusions and is justified by the fact that it has been demonstrated that for horizontal fractures with radii less than four-thirds of the depth, the solution for infinite and semi-infinite medias are essentially identical.[10] Since the turbulent region extends only a few feet from the well, its additional contribution to the average pressure (P_a) can be neglected. The average pressure can now be computed from Eq. (11) by observing that h_f = 0 at the tip of the fracture (r = c), according to

$$P_a = \frac{\pi r'^2 h_f + 2\pi h_f \int_{r'}^{c} r\,dr - \frac{2\mu Q}{24\ \pi\alpha^3} \int_{r'}^{c} r \ln \frac{r}{r'}\, dr}{\pi c^2} \tag{13}$$

which reduces to

$$P_a = \frac{\mu Q}{48\ \pi\alpha^3} (1 - r'^2/c^2)\ . \tag{14}$$

Since c will always be much larger than r', the value of the quantity in brackets will be approximately unity and can be dropped. The geometry of an ellipsoid gives the relationship between the maximum width (W_{max}) and the average width (α) in a direction perpendicular to an axial plane:

$$W_{max} = 3\,\alpha/2 \ . \tag{15}$$

Equations (11), (12), (14), and (15) can now be combined to yield the expressions

$$h_f = \frac{\sqrt{3}}{8} \ln \frac{c}{r'} \, [E/(1-\nu^2)c]^{3/4} \, [\pi^2 \mu Q]^{1/4} \tag{16}$$

$$W_{max} = \frac{\sqrt{3}}{2} \, [(1-\nu^2)c\mu Q/\pi^2 E]^{1/4} \ , \text{ and} \tag{17}$$

$$c = [9 \, E \, \Delta^4/\pi^2(1-\nu^2)\mu Q]^{1/9} \tag{18}$$

where

Δ = cumulative volume injected (ft^3).

It should be recognized that the center thickness of the fracture, defined by Eq. (17), is the theoretical separation of the two sides of the fracture at the well due to the internal fluid pressure. The actual thickness of the slot at the well and for a short distance inside the fracture will be much larger as a result of the mechanical erosion of both the sand-jet slotting operation and the high velocity fluid flows at that point. The variation of laminar zone losses (h_f) and the fracture radius (c) as a function of the cumulative volume injected (Δ) as expressed by Eqs. (16) and (18) is shown on Fig. 2. The point at which h_f is evaluated (that is, at r') varies as a function of the flow rate (Q) according to Eq. (7). This variation acts to depress the curves at the higher flow rates, which would otherwise be parallel to and lie above the curve for Q = 40 gpm.

The evaluation of the losses in the turbulent region (h_e) is much more difficult. In relatively simple hydraulic systems, such as the fluid in a tank exiting through a small horizontal pipe, the entrance losses can be defined by

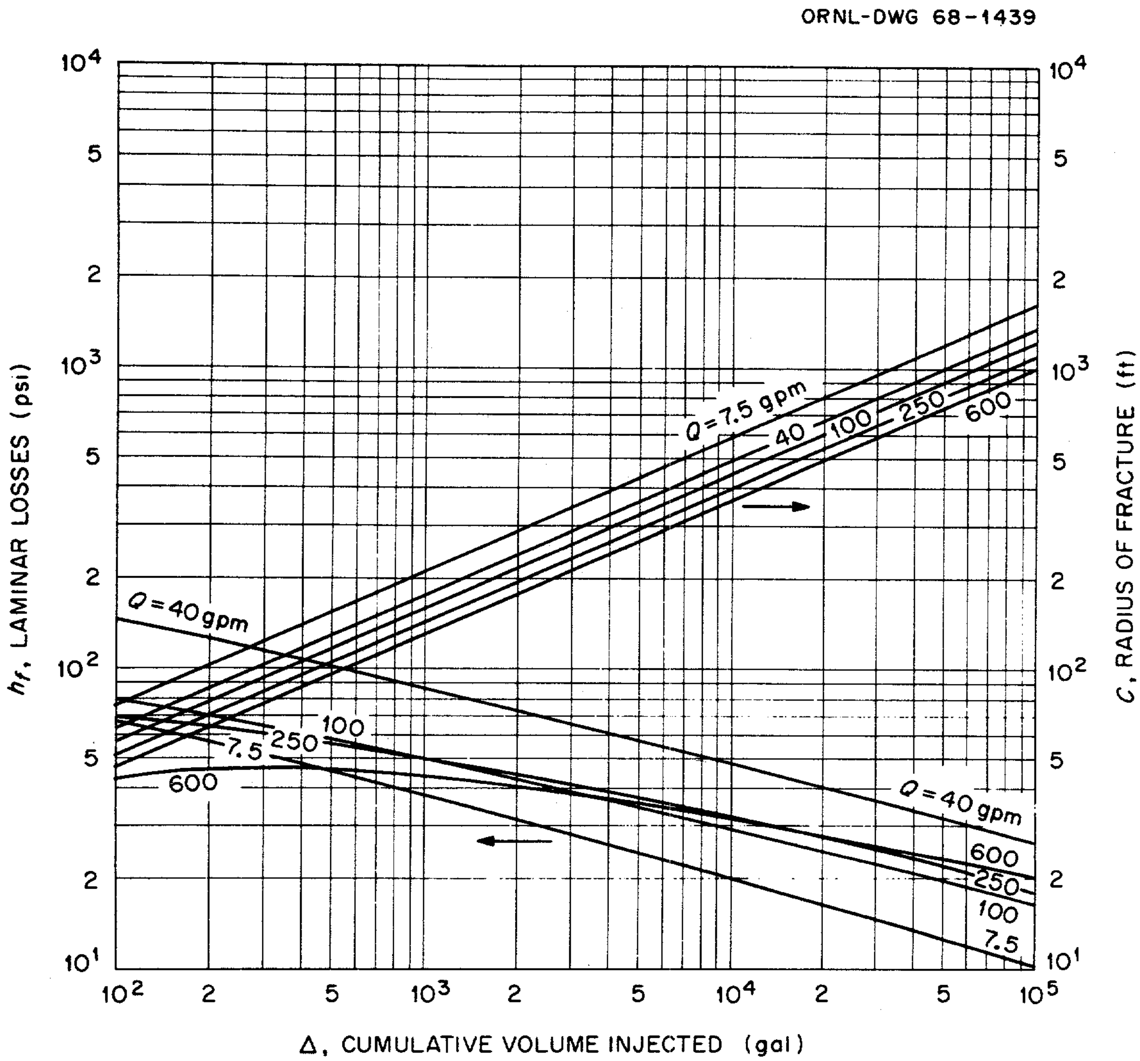

Fig. 2. Laminar Flow Zone Losses (h_f) and Fracture Radius (c) as a Function of Cumulative Volume Injected.[1]

$$h_e = K_e(\gamma V^2/2\ g) \tag{19}$$

where the coefficient K_e has a value between 0 and 1.0, depending on the geometrical configuration of the entrance pipe. The value of K_e is known only for the very simplest systems, and usually it is necessary to evaluate K_e by experiment. In hydraulic fracturing, it is impossible to make a reasonable estimate of the shape of the fracture just outside the well casing. Even if the value of the entrance loss coefficient (K_e) could be estimated, there would remain the problem of selecting the proper velocity (V) to use in Eq. (19). The difficulty in selecting a velocity results from the relationships expressed in Eq. (5). In the region very close to the well casing, the thickness of the fracture (α) may decrease more rapidly than the radial distance (r) increases, because of the physical erosion of the shale. In this case, the maximum bulk velocity of the fluid in the fracture will occur at some short distance inside the fracture rather than at the casing slot. One way around these difficulties is to assume that the proper value of the velocity for use in Eq. (19) occurs at some characteristic distance from the well for each flow rate. It is then possible to write the generalized relationship:

$$V = AQ^m \Delta^n \tag{20}$$

where A is an arbitrary constant and the Δ^n component grows out of the fact that part of the thickness of the fracture [α in Eq. (5)] is a function of the total volume injected (Δ). Substitution of Eq. (20) into Eq. (19) produces the expression

$$h_e = \frac{\gamma}{2\ g} B\ Q^{2m}\ \Delta^{2n} \tag{21}$$

where

$$B = A^2 K_e \ .$$

The coefficients, B, m, and n, can now be evaluated from an experimental water injection carried out over a sufficient range of both flow rates (Q) and total injected volume (Δ), provided the fracture produced is approximately horizontal and circular.

Such a test was conducted at the Oak Ridge facility in December of 1967 for the expressed purpose of obtaining data on site-testing procedures. At this site, there is sufficient evidence,[7] in the form of cores and well logs related to previous grout injections, to justify the assumption that the resulting fractures are approximately circular and horizontal. This test injection was made through a new slot cut in the well casing at a depth of 852 ft below the surface. A total of 44,700 gal of water was injected at various flow rates and at the pressures shown on Fig. 3. At this injection well, the fluids (water in this case) are pumped down a 2.44-in.-ID tubing string suspended in the 4.67-in.-ID well casing to a point about 25 ft above the slot. The injection pressures given on Fig. 3 were monitored at the surface in the static annulus between the inner tubing string and outer well casing. Therefore, these pressures must be corrected only for the difference in elevation between the surface gauge and the underground slot in the well casing. The friction losses in the tubing string, which are appreciable, are not registered by this gauge, and the losses in the 25-ft length of casing are negligible. Flow rates of 40, 100, and 250 gpm were maintained in sequence for periods of 10 to 40 min each. These periods permitted the flow transients (See Section IV.) to be largely eliminated. The analysis was consequently carried out using the last points (solid dots on Fig. 3) of each constant flow-rate period. These data are given in Table 1. The empirical expression obtained by least squares curve fitting was

$$h_e = \frac{\gamma}{2\ g} \left(1.1175 \times 10^6\right) Q^{2(0.07)} \Delta^{2(-0.09)} \ . \qquad (22)$$

This equation and the experimental points are shown on Fig. 4.

It is advantageous to introduce at this point the concept of a "pumping characteristic," which is simply the manner in which the measured pressure in the well varies throughout the course of the injection, for a given flow rate. The best method of presenting this data for interpreting differences in the fracture orientation and shape is to plot the excess of the measured pressure above rock overburden pressure ($P_w - \sigma$) as a function of the total volume injected (Δ) for each flow rate (Q) on a log-log graph. (See Fig. 6, for example.)

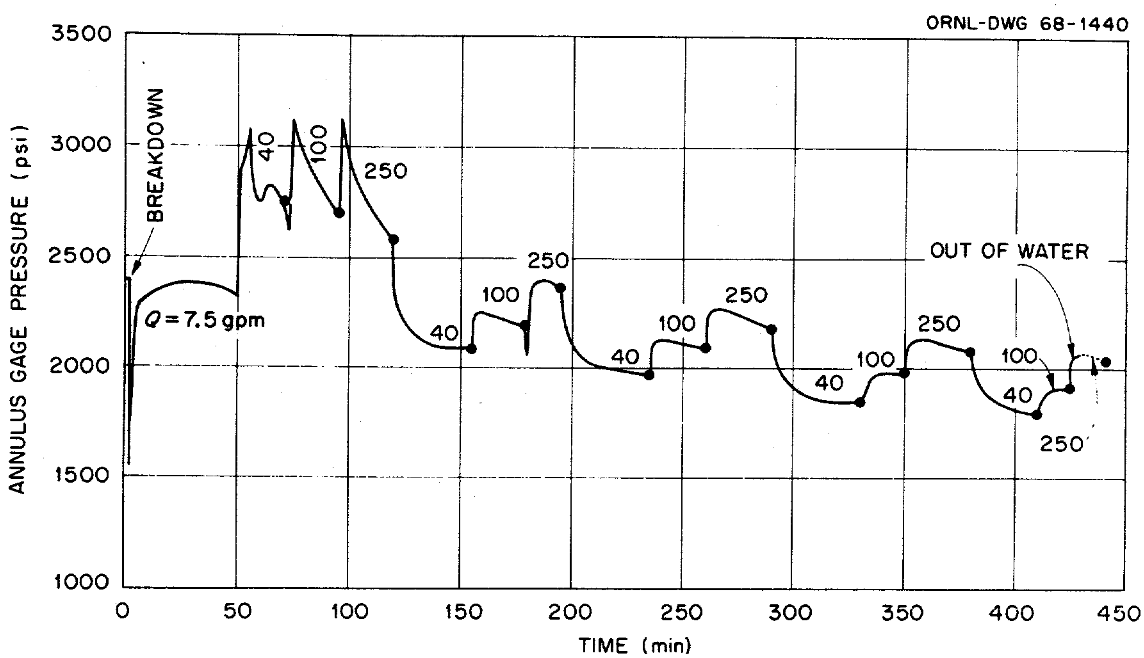

Fig. 3. Pressure, Flow-Rate, Time History of Water Injection Test, December 1967.

Table 1. Significant Data Points from Water Injection Test

Time (min)	Q (gpm)	Δ (gal)	Gauge Pressure (psi)
72	40	1,305	2,750
95	100	3,508	2,700
120	250	9,580	2,590
155	40	10,980	2,090
175	100	13,850	2,200
195	250	17,530	2,370
235	40	19,290	1,980
260	100	21,870	2,100
290	250	29,300	2,180
330	40	31,300	1,850
350	100	33,240	1,980
380	250	40,810	2,080
410	40	42,280	1,800
424	100	43,640	1,920
(440)	250	(47,700)	(2,040)

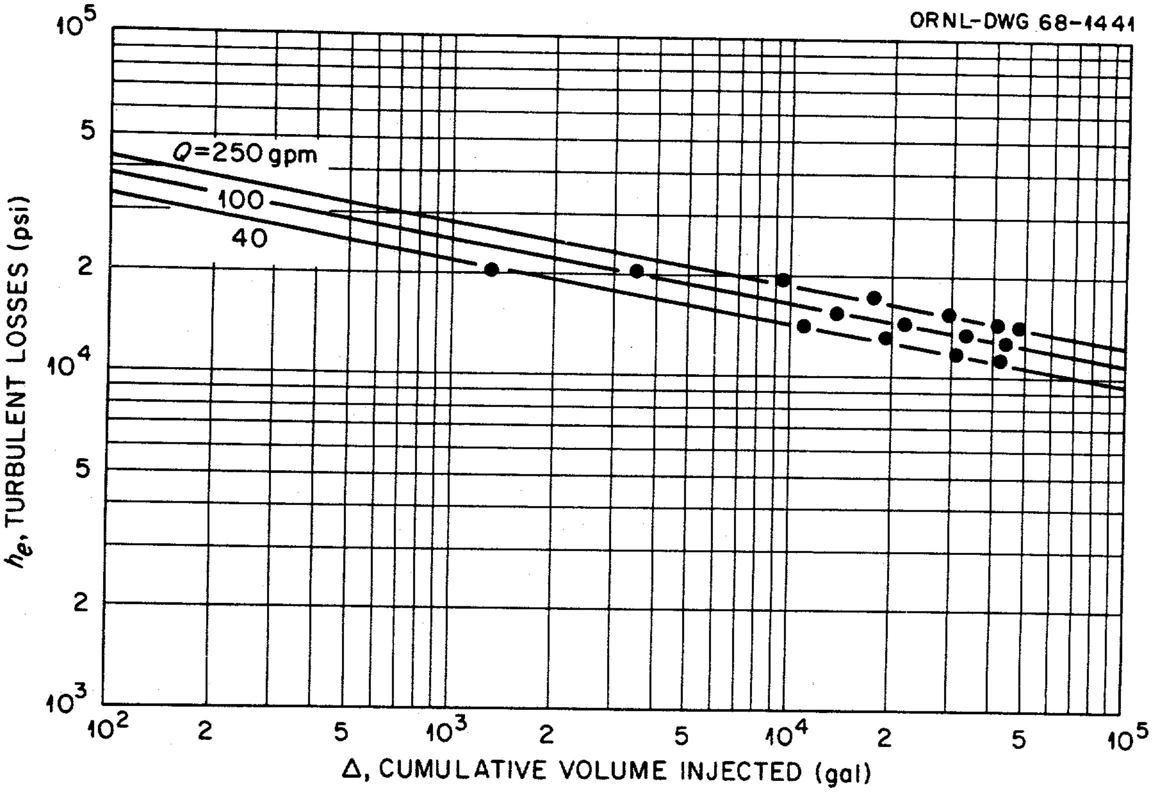

Fig. 4. Turbulent Flow Zone Losses (h_e) as a Function of Injected Volume as Determined from Experimental Water Injection Test.

The pumping characteristic for a horizontal circular fracture will then be sum of Eqs. (16) and (20) plotted in this manner.

III. NONCIRCULAR HORIZONTAL FRACTURES

There is some evidence from the numerous experimental injections carried out in the Oak Ridge area that a fracture may sometimes be propagated preferentially in one direction away from the injection well, producing a grout sheet which is elongated in plan instead of circular. This fracture shape can be analyzed if it is assumed that the fracture is confined in the horizontal plane between two parallel boundaries. In this situation, the fracture will first expand circularly until the limit lines are reached; thereafter, it will extend only in the directions parallel to these restraining lines. If the restricted horizontal dimension of the fracture is defined as ψ, the analysis of the behavior of a water test in this configuration can proceed in a manner similar to the previous section. First, with the injection well at the center and with the limit of the turbulent zone much less than the restricted fracture width ($r' << \psi/2$), the turbulent zone losses (h_e) should be very nearly the same as for a circular fracture. The laminar zone losses will be quite different however, since the nature of the fluid flow in the fracture has changed considerably. Since the flow is in only two directions from the injection well, the bulk velocity of the fluid will be

$$V = 2\ Q/\pi\psi\alpha \tag{23}$$

The flow cross section can be approximated by a very flattened ellipse, where[10]

$$R_H = \alpha/2.546. \tag{24}$$

Substitution of these expressions into Eqs. (4), (8), and (9), which otherwise remain the same, produces the differential equation

$$dP = \frac{(2.546)^2\ \mu Q}{24\ \pi\psi\alpha^3}\ dr \tag{25}$$

Integration of this equation gives the pressure distribution in the fracture

$$P_{(r)} = \frac{(2.546)^2}{96\ \pi\psi\alpha^3} (c - r) \qquad (26)$$

where

$r' \leqq r \leqq c$.

Sneddon[11] gives the thickness of this type of restricted fracture at any point as a function of the distance from the center as

$$\alpha = \frac{2(1 - \nu^2)\psi\ P_{(r)}}{E} \qquad (27)$$

These last two equations provide an estimate of the fracture thickness (α) as a function of r. Substitution of this relationship into Eq. (25) and integration between the limits of r' and c then gives an estimate of the laminar losses (h_f).

$$h_f = 1/\psi \left[\frac{4(2.546)^2\ E^3\ \mu Q(c - r')}{3\ \pi(1 - \nu^2)^3} \right]^{1/4} , \qquad (28)$$

where, from the geometry of an ellipsoid,

$$c = 3 \left[\frac{2^2\ E\ \Delta^4}{\pi^3\ \psi^4\ \mu Q(1 - \nu^2)(2.546)^2} \right]^{1/5} , \text{ and} \qquad (29)$$

$$W_{max} = \left[\frac{(1 - \nu^2)(2.546)^2\ \mu Qc}{12\ \pi E} \right] \qquad (30)$$

The variation of the laminar zone losses (h_f) and the half length of the fracture (c) for this case, assuming a restricted width of 200 ft, are shown in Fig. 5 as a function of the cumulative volume (Δ). If the injection well is located near one end of the elongated fracture, the bulk velocity in Eq. (23) is doubled which results in a friction loss (h_f) which is twice as large as indicated in Eq. (28).

This analysis of noncircular fractures is somewhat less rigorous than the preceding deviations for circular fractures, primarily because

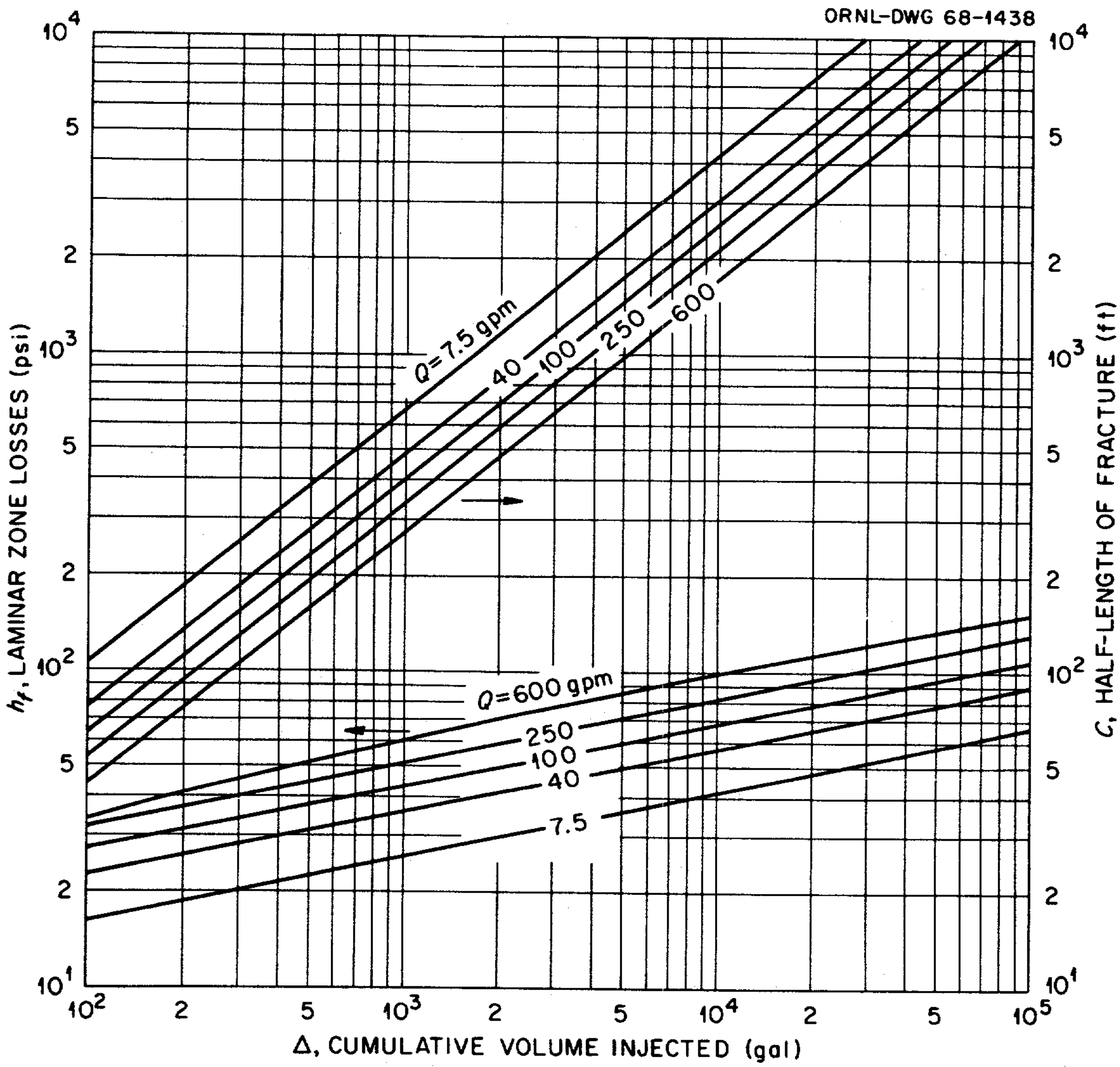

Fig. 5. Laminar Flow Loss (h_f) and Fracture Half-Length (c) as a Function of Injected Volume (Δ) for a Horizontal, Noncircular Fracture.

of the reduced symmetry. It should, however, be sufficient for the purposes of this report, since it indicates that there is a gross difference in the behavior of the pressure function in the laminar zone between the two cases of circular and noncircular fractures. Since the turbulent zone losses should be approximately the same as for a circular fracture, the pumping characteristic for horizontal noncircular fractures can be obtained as the log-log plot of the sum of Eqs. (22) and (28).

IV. MULTIPLE FRACTURES

Cores retrieved from the injection horizon at the Oak Ridge site frequently show that the grout has been deposited in several parallel fractures and presumably this also could occur with water injections. If it is assumed that the branching occurs at or near the well and that a channel is maintained in such a way that the injected fluid is divided equally between the subfractures, a set of "scaling" laws can be derived for modifying the laminar flow loss equations. For a horizontal circular fracture, Eqs. (16), (17), and (18) are modified by

$$\begin{aligned} \Delta' &= \Delta/n \\ Q' &= Q/n \\ C' &= C/\sqrt[3]{n} \\ W'_{max} &= W_{max}/\sqrt[3]{n} \\ h'_f &= \left(\frac{\ln c'/r'}{\ln c/r'}\right) \end{aligned} \qquad (31)$$

where n is the number of multiple fractures and the primed parameters apply to the multiple fracturing case, while the unprimed parameters are those for an equivalent single fracture.

If some sample values are used, it is seen that the multiple fracturing reduces the laminar loss component (h_f) by approximately 3% for n = 2 to 9% for n = 6. These factors may vary slightly around these mean values depending on the value of c. Since the change is relatively small, the effect of multiple fractures on the laminar losses can be considered to be negligible even though the dimensions of the fracture may be significantly altered.

It is also possible that the formation of multiple fractures may significantly affect the turbulent zone losses (h_e). This influence would be seen as a gross change in the value of the coefficients B, n, and m in Eq. (21). Since these coefficients are evaluated empirically and there is, at present, only one set of experimental data, it is difficult to assess how much and in what direction the value of these coefficients will be changed. Multiple fracturing may, however, be the explanation of some of the differences discussed in Section VI.

V. NONHORIZONTAL FRACTURES

In order to provide the background required to understand and appreciate the discussion of nonhorizontal fractures, it is necessary to digress briefly into the realm of fracture mechanics as applied to the development of hydraulic fractures. The analysis so far has considered only a situation which might be called dynamic equilibrium as far as the rocks around the fracture are concerned. In this case, the dynamic pressures, that is, the fluid pressures in the fracture required to maintain flow against the various losses, are balanced by elastic stresses in the rock around the fracture. When pumping is stopped, the flow velocities, and consequently the hydraulic losses, will decrease very rapidly. The fluid pressure in the fracture will no longer be able to maintain the fracture thickness against the elastic stresses in the rock. Some of this stored elastic energy will therefore be recovered, causing the fracture to become thinner and larger between the time the pumps are stopped (with the well shut-in) and the time a "static" equilibrium is attained. This equilibrium or minimum fracture extension pressure (P_o) has been derived by Sack[12] for a circular crack,

$$P_o = [\pi\lambda E/2(1 - \nu^2)c]^{1/2} \qquad (32)$$

where

λ = specific surface energy of the rock and

P_o = static pressure inside the fracture in excess of the overburden or confining pressure.

For any reasonably sized fracture (say, $c > 100$ ft), the value of the minimum fracture extension pressure is very small ($P_o < 10$ psi) and it has consequently been neglected in all of the previous analysis and most of that which follows, although it is understood that it does contribute a small amount to the pressure measured at the well. The total fluid pressure in the fracture is, therefore, the sum of a static element equal to the overburden pressure (σ) where this portion merely supports the weight of the overlying rock, a small component required to drive the fracture into the virgin rock (P_o), and the viscous losses. These last two elements of pressure combine to elastically deform the rocks and are thus responsible for producing the width of the fracture.

At any place on the tip of the propagating fracture, the viscous losses are reduced to zero and the fluid pressure is just equal to the overburden plus the minimum fracture extension pressure. If the overburden is less at some points around the crack periphery than at others, the fracture will be propagated only into those low confining stress areas. This feature is actually a corollary to the fundamental principle that the fracture will form in a direction perpendicular to the direction of the least compressive stress and can be stated more exactly as: within the plane (or surface) defined by the least stress direction, the fracture will be propagated preferentially towards those regions where the stress perpendicular to the fracture (the minimum principal earth stress) is itself lowest. For example, if the minimum principal stress is everywhere vertical, a horizontal hydraulic fracture will be developed. But, if the value of this vertical stress varies throughout the horizontal plane, being lower say to the north than to the south, the fracture will extend itself to the north. In this respect, the fluid in the fracture behaves somewhat like water seeking its lowest level. If the values of the vertical stress in the horizontal plane were to be contoured, the analogy to the flow of surface water over topographic contours is even more valid.

It is now possible to consider the behavior of several hypothetical situations where a horizontal fracture is developed but encounters an overburden stress which is not constant. For this discussion, it will be assumed that the overburden stress varies as some function of distance from the well, but that this function is axially symmetrical about the injection well.

Case I. Overburden stress increasing as a function of r. In this case Eq. (2) can be written

$$P_w = \sigma_w + h_e + h_f + \delta\sigma_c \tag{33}$$

where

σ_w = overburden pressure at the well and

$\delta\sigma_c$ = the amount of overburden pressure at the crack tip in excess of σ_w (that is, $\delta\sigma_c = \sigma_c - \sigma_w$).

Because of the increasing function of $\delta\sigma_c$, the injection pressure will increase throughout the injection. This added component of static pressure is balanced by an added component of rock strain energy, causing the fracture to widen. However, since the additional increment of stored energy is due to a static pressure (not dynamic), it is not recoverable when the pumps are stopped.

Case II. Overburden stress decreasing slightly as a function of r. Here, the pressure equation will be

$$P_w = \sigma_w + h_e + h_f - \delta\sigma_c \; . \tag{34}$$

The increment of static pressure ($\delta\sigma_c$) is now subtracted from the pressure holding the fracture open, causing it to reduce its width. However, this reduction in thickness will cause a compensatory increase in the dynamic losses h_e and h_f which prevent the well pressure (P_w) from decreasing as rapidly as $\delta\sigma_c$ decreases during the injection. If the total change in overburden pressure ($\delta\sigma_c$) is less than the minimum fracture extension pressure [P_o in Eq. (31)], the fracture will be stable after pumping has stopped and it has extended until the recoverable stored strain energy has been utilized.

Case III. Overburden stress decreasing rapidly as a function of r. The pumping phase of this case will be the same as Case II, because the decrease in crack thickness will be overcompensated by the increase in the friction losses, thus maintaining the continuity of the fracture. However, the situation is metastable and as soon as the pumping stops, thereby eliminating the loss components (h_e and h_f), the fracture will begin migrating away from well, because the difference in the overburden pressures ($\delta \sigma_c$) is now greater than the minimum fracture extension pressure.

The behavior of nonhorizontal fractures could now be predicted, if the original state of stress in the earth at all points along the fracture were known, simply by substituting the words "stress perpendicular to the fracture" for "overburden stress" in the above discussion and correcting for the elevation change [δz in Eq. (1)]. It seems reasonable to assume that the vertical component of earth stress is equal to the weight of the overlying material

$$\sigma_z = \gamma_R H \tag{35}$$

where

H = depth below surface.

However, there is only very limited experimental information on which to base an assumption of the horizontal component.[5] The usual assumptions concerning the horizontal component of original earth stress are that:

1. the natural plasticity of the rocks over the geologic time available has relieved any shear stresses, thereby resulting in a hydrostatic stress condition where

$$\sigma_z = \sigma_x = \sigma_y \text{ , or} \tag{36}$$

2. the rocks at depth behave elastically in response to the weight of the overlying material so that

$$\sigma_x = \sigma_y = \frac{\nu}{1 - \nu} \sigma_z \tag{37}$$

Neither of these assumptions is valid for any location where hydraulic fracturing produces horizontal fractures, because they both lead to the situation where the least compressive stress is not oriented vertically.

Waste disposal by hydraulic fracturing is unlikely to be attempted at depths greater than 3000 ft nor less than about 1000 ft. Furthermore, any proposed site will undoubtedly contain features suggesting the presence of a compressive horizontal tectonic stresses such as thrust faulting and folding. If consideration is limited to these situations, it is possible to postulate a generalized expression for the state of stress in the earth's crust where

$$\sigma_z = \gamma_R H$$
$$\sigma_x = \sigma_y = S + KH \qquad (38)$$

where S and K are undefined constants. This is equivalent to saying that the horizontal earth stress is made up of a constant tectonic component (S) plus a component which varies linearly with depth (K). The coefficient K may come from Eq. (36), giving $K = \gamma_R \nu/1 - \nu$. At least a part of the value of K may also come from a (assumed linear) variation with depth of the horizontal tectonic earth stress. Because of this possible tectonic contribution, K might have any value.

In this discussion, a tacit assumption that σ_x, σ_y, and σ_z are principal stresses has been made. However, if a hydraulic fracture is produced which is neither horizontal nor vertical, the implication is that the principal stress axis is inclined. In this case, σ_x, σ_y, and σ_z are not principal stresses. If the dip of the hydraulic fracture is θ and the principal stress which is oriented perpendicular to the fracture plane is defined as σ_θ, the value of σ_θ will be given by resolving the three orthogonal stresses and the shear stresses. The determination of σ_θ in this way is not only difficult, since the value of the shear stresses is unknown, but also unnecessary, since (if these shear stresses are also linearly related to the depth) it is possible to express the value of σ_θ in terms of undefined coefficients similar to Eq. (38),

$$\sigma_\theta = S' + K' H \qquad (39)$$

An equivalent but more useful statement is

$$\sigma_{\theta_c} = \sigma_{\theta_w} - K' \delta z \tag{40}$$

where

σ_{θ_w} = earth stress perpendicular to an inclined fracture measured at the depth of the injection,

σ_{θ_c} = earth stress perpendicular to the fracture at its tip,

K' = variation of this confining stress with respect to a change in elevation, and

δZ = vertical distance above or below the elevation of the slot. (Up is positive.)

Using this definition [Eq. (40)] of the earth stress and its variation with depth, it is possible to write a generalized expression for the fluid pressure measured at the injection well

$$P_w = \sigma_{\theta_w} + h_e + h_f + \delta z(\gamma - K') \tag{41}$$

Since the terms h_e and h_f are related to the volume of fluid injected (Δ), and δz is also a function of Δ because $\delta z = c \sin \theta$, it is now possible to examine the behavior of inclined fractures and their pumping characteristics over a range of values of K' using Eq. (41).

Case I - K' = 0. In this case, there is no variation of the confining stress (σ_θ) as a function of depth. This probably represents a lower limit of K', since it is difficult to imagine a situation where K' would be negative; that is, where the confining (or horizontal) stress would become less with increasing depth. Since the fluid pressure in the fracture at its lowest point is greater than that at any other point by an amount equal to the static head ($\gamma \delta Z$), the fracture will be propagated downward and will undoubtedly be non-circular. The pumping "characteristic" of this type fracture will be

$$P_w - \sigma_{\theta_w} = h_e + h_f + \gamma \delta Z \text{ or}$$
$$P_w - \sigma_{\theta_w} = h_e + h_f - \gamma c \sin \theta \tag{42}$$

where

θ = dip angle (measured from horizontal) of fracture and

h_f = Eq. (28) with ψ a very small number.

This fracture will become self-propagating if $\gamma \delta Z > P_o$.

Case II - $0 < K' < \gamma$. As in Case I, the nonhorizontal fracture for these values of K' will be propagated preferentially downward, with this tendency decreasing as K' approaches γ. The pumping characteristic is obtained directly from Eq. (41).

Case III - $K' = \gamma$. In this case, the vertical differential of the gradient of the confining stress is equal to the density factor of water (0.433 psi/ft). Therefore, these two effects cancel out of Eq. (41) and the fracture "sees" only a constant confining stress. Its behavior and pumping characteristics will be therefore exactly the same as for a horizontal circular fracture:

$$P_w = \sigma_{\theta_w} = h_e + h_f \qquad (43)$$

Case IV - $K' = \gamma_R$. For a vertical fracture ($\theta = 90^\circ$), this case reduces to the assumption of complete plastic relaxation of the rocks as defined by Eq. (36). Since the confining pressure at the top of the fracture is less than at the bottom, even after compensating for the static head of water in the fracture, a noncircular fracture will be propagated upward. The pumping characteristic is then

$$P_w - \sigma_{\theta_w} = h_e + h_f + \delta Z(K' - \gamma) \text{ or}$$

$$P_w - \sigma_{\theta_w} = h_e + h_f + (K' - \gamma)c \sin \theta$$

The fracture will continue to propagate upward even after the pumps are stopped if $\delta Z(K' - \gamma) > P_o$.

Case V - $\gamma < K' < \gamma_R$. In this case, the nonhorizontal fracture changes from probably circular, as in Case III, to upward as in Case IV, as K' approaches γ_R. Equation (44) will apply throughout as the proper expression of the pumping characteristic.

Case VI - $K' > \gamma_R$. This case represents all situations where the confining (or horizontal) stress increases rapidly with increasing depth. Since so little is known about the tectonic component of the horizontal stress, it is difficult to estimate what might be an upper limit of K'. For hydraulic fracturing, however, the larger the value of K' the more strongly the fracture will propagate upward in preference to other directions.

The predicted pumping characteristic for a water injection at 100 gpm with the extreme case of vertical fracturing ($\theta = 90^{\circ}$) under the situations described in Case I [Eq. (42)], Case III [Eq. (43)], and Case IV [Eq. (44)] is shown in Fig. 6. Case III ($K' = \gamma$) is also the pumping characteristic for horizontal circular fractures where σ_{θ_w} is then equal to the overburden pressure. As suggested by this figure, it should be possible to distinguish vertical or inclined fractures from horizontal ones on the basis of pumping characteristic curves, provided that K' is sufficiently different from γ. When K' is approximately equal to γ, the situation is ambiguous and it is not possible to distinguish between vertical and horizontal fractures on the basis of the pumping characteristic alone.

However, there is an additional item of information available from a water-injection test--the value of σ_{θ_w}. The value of this component of the earth stress controls the orientation of the fracture, since, for vertical fractures, it must be less than the vertical overburden pressure; whereas, for horizontal fracture, σ_{θ_w} must be greater than the weight of the overlying material. It is the value of σ_{θ_w} which one attempts to measure as the instantaneous shut-in pressure or as a component of the breakdown pressure, with all of the difficulties associated with these measurements. A water-injection test should at least provide an additional measure of σ_{θ_w}, therefore further contributing to an overall interpretation of the fracture orientation, even when the pumping characteristic is otherwise ambiguous.

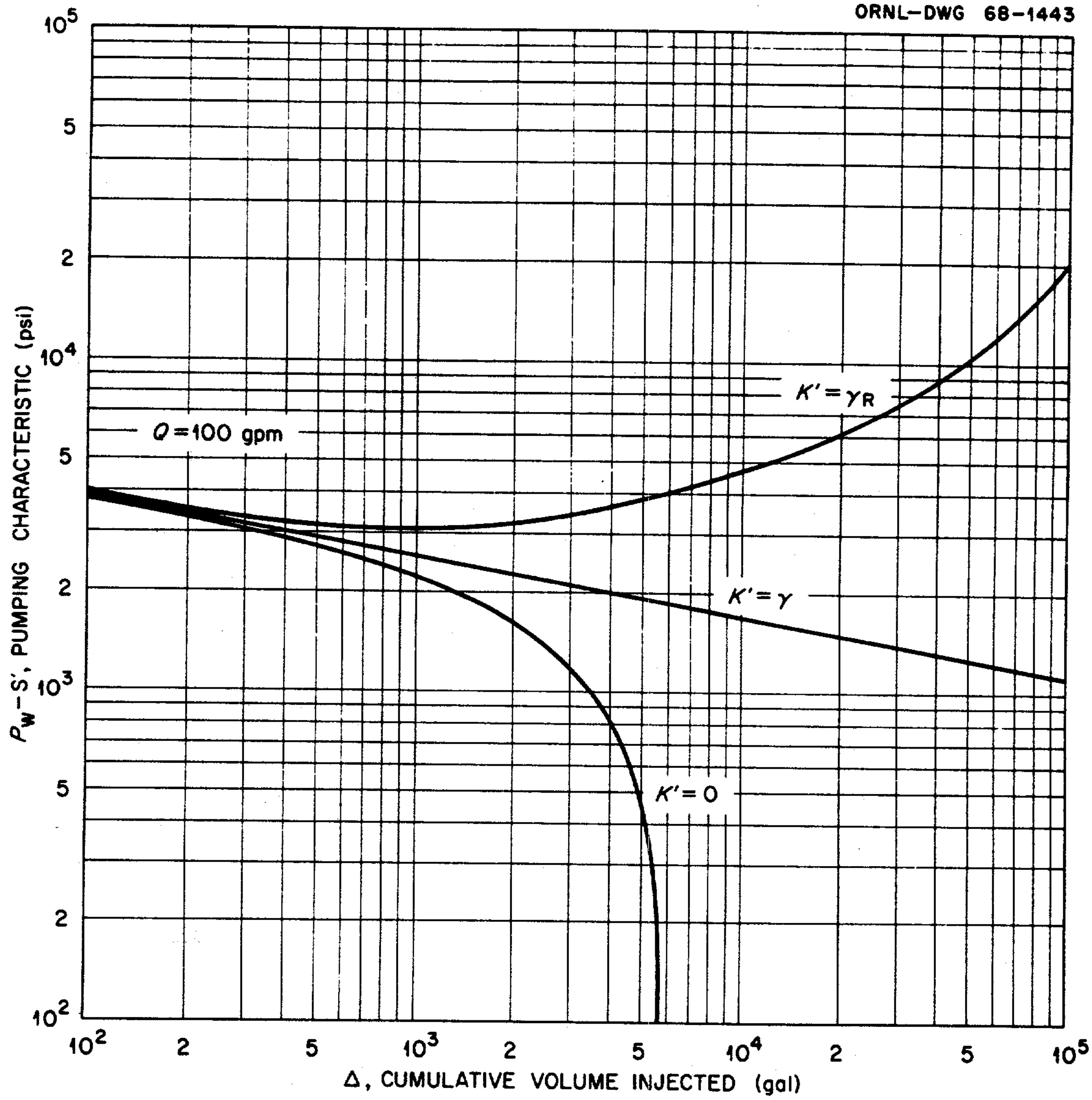

Fig. 6. Pumping Characteristics for Vertical Fractures.

VI. RECOMMENDATIONS

The weakest link in this analysis is the evaluation of the turbulent losses (h_e) which was empirical and based on experimental data from one test injection. In all probability, the value of the coefficients defining these losses [See Eq. (21).] will be different for each slot in the well casing and even for each injection into the same slot. This is because their value is so highly dependent upon the geometry of the slot and the first few feet of fracture. It should be emphasized that these reservations do not detract from the validity of the rest of the analysis, but they will affect the value of the pressure on, for example, Fig. 6. The possible variation in the determination of the turbulent losses can be illustrated by examination of the only comparable data available--the injection records for the water tests carried out at this same site prior to the experimental grout injections. The pumping characteristic for the "50,000-gal test" and for the "23,000-gal test" are shown in Fig. 7. Both of these injections should have been more or less horizontal and circular, and the curves show the exponential decrease in pressure with injected volume predicted for that situation. However, the value of the pressure for the 50,000-gal test is about 800 psi below that predicted on Fig. 6 and about 500 psi below the predicted value for the 23,000-gal test. Also, notice the difference in the values of the pressure characteristic between these two tests, even though they were carried out in the same slot.

One possible way to avoid this difficulty in a site examination would be to locate one of the "observation" wells (which probably would have been drilled as a geologic exploration well) very close to the injection well, say about 25 ft. The measurement of pressure in this open well, after the fracture had intersected it, would then eliminate the turbulent zone losses and their associated difficulties from all further interpretation. In addition to eliminating one of the major unknown and uncontrollable variables, this technique would add appreciably to the sensitivity of the technique of interpreting the fracture orientation. This increased sensitivity is gained because one of the larger components in the pumping characteristic equations (h_e) is removed, making much smaller variations in the other components detectable.

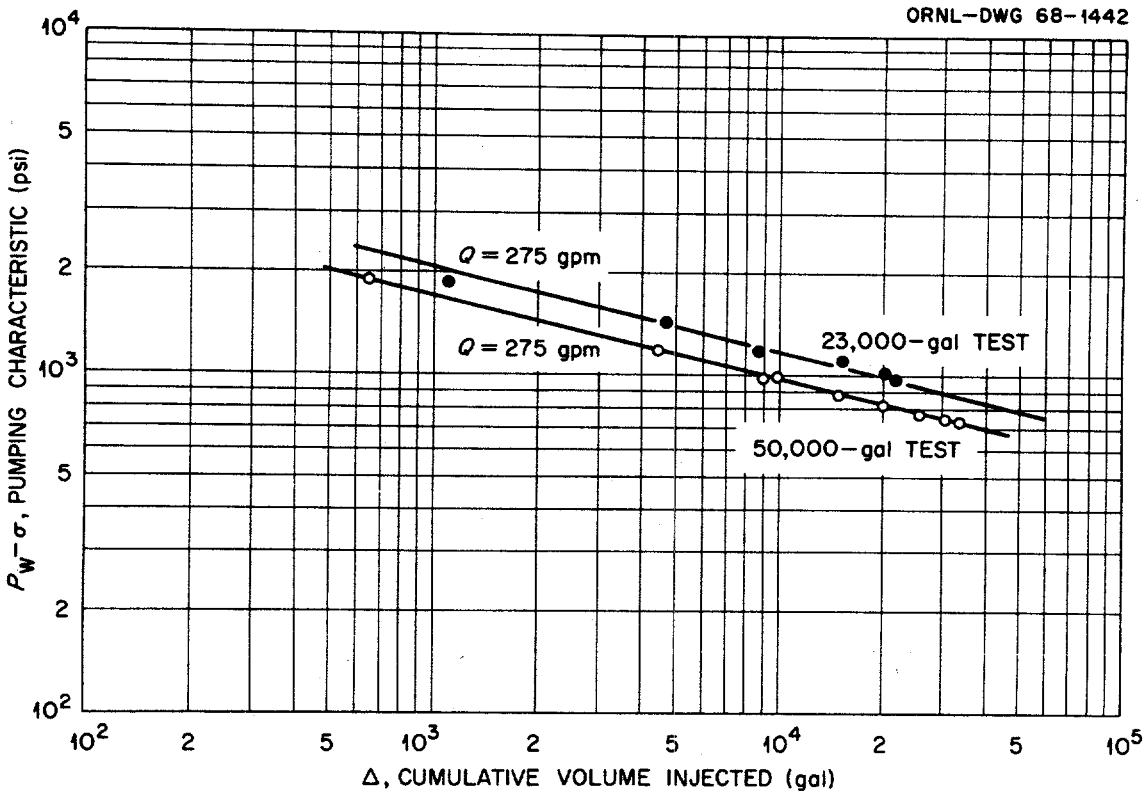

Fig. 7. Experimental Pumping Characteristics from Preliminary Water Injection Tests at ORNL.

As an alternative solution to the difficulties with the turbulent zone losses, an attempt could be made to at least establish the range of possible values for several geometric configurations based on scaled laboratory experiments. Although this solution is not as direct as the previous one, it would appear to be capable of providing some useful information at very modest cost.

It has been shown that the pressure history of water-injection tests provides a considerable amount of information concerning the orientation of the fracture. This same information is of some interest for each waste slurry injection, since it would provide an additional control over the safety of the waste-disposal technique. Therefore, in spite of the formidable analytical difficulties related to the non-Newtonian properties of these slurries, an attempt should be made to broaden this analysis to include these fluids. This broadening would then encompass a much larger body of experimental data which has been obtained from the slurry injections already conducted at the Oak Ridge facility.

VII. CONCLUSIONS

The analysis presented in this report indicates that there is a considerable amount of information directly related to the orientation of fractures contained in the pressure history of the injection itself. The interpretation of this pumping pressure history should now be possible, based upon this analysis and the increased understanding of the mechanisms which it provides, in spite of the present weaknesses. For these reasons alone, a preliminary water injection would appear to be an advantageous test to conduct early in the testing program at any proposed hydraulic fracturing waste-disposal site. There are, of course, other advantages, related to the measurement of breakdown and especially shut-in pressures, but these were excluded from consideration in this report.

The procedure for this site evaluation, water-injection test should be similar to the experimental injection conducted recently at Oak Ridge; that is:

1. the injection rate should be varied sequentially between three (or even four) different flow rates with each rate held constant for 20 to 40 min,
2. a total volume approaching 100,000 gal should be injected,
3. careful measurements of the flow rate, total volume injected, and pressure should be made. The pressure measurement should be made in a static annulus (as at ORNL), with a downhole pressure transducer at the elevation of the slot, or as discussed in the previous section, in a nearby observation well.

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