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Report No.: A-3685
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Date: February 12, 1947

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THE PRECISION OF THE CASCADE MATERIAL BALANCE ON T AND X AT K-25

Carbide and Carbon Chemicals Corporation Operating Contractor for the U.S. Atomic Energy Commission.

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B. Carter

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ENGINEERING DEVELOPMENT DIVISION

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Report No.: A-3685
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To: S. Cromer

Date: February 12, 1947

THE PRECISION OF THE CASCADE MATERIAL BALANCE ON T AND X AT K-25

Written by: C. Daniel
B. Carter

Abstract

This report is a revision of Section VII of the Report No. 2.16.13, "Accounting for T and X in the Diffusion Plant", by Benedict, Daniel, and Squires, (4/8/46) to conform to new conditions of operation. The required precisions and accuracies of T-analysis, X-assay, and weighings, are given, together with the required number of measurements of each type.

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Summary

Equations are derived relating the precision and accuracy of each type of measurement to the precision of the overall material balance on K and U. The premises of this report differ in several important respects from those of 2.16.13.

a. The cascade operating conditions are widely different.

b. The annual tolerance on unaccounted-for K is different and differently interpreted.

c. Some quantities are measured differently, e.g. weights of feed and waste.

d. Some improvements in logical method have been made, e.g. chemical purities are treated here at the same time as other variables, instead of as later corrections.

e. Allowance is made for sampling errors.

f. All precisions and some biases formerly guessed are now known.

It is concluded that the precisions now being attained in U-analysis, K-assay and weighing are sufficient and that required precisions of constants (i.e., the uncertainties in systematic errors or biases) are only very slightly changed from those formerly recommended.

Actual values of required precisions and accuracies for all stream variables are given in Table II. The precisions required for inventory variables and constants will be treated in a later report.

Introduction

The methods used for propagating the effects of random errors are well-known, having been fully understood by K. F. Gauss (1). The method developed in 2.16.13, and extended in this report, of propagating the effects of systematic errors is new, so far as the present writer is aware, and therefore should be criticised freely.

Part I of this report shows how the tolerance limit given for annual unaccounted-for K is translated into variance-assignments to the various parts of K-25-27.

Part II shows how the target variances set for each K-stream and K-inventory can be met, by taking a sufficient number of measurements of known precision for each type; and by determining constant parameters with known precisions.

---

(1) "Theoria Combinationis Observationum erroribus minimis obnoxiae", K. F. Gauss, Göttingen, 1827.

Part I - Maximum Allowable Annual Unaccounted-for X

The (unofficial) value set for the maximum permissible loss per year is 20 Kg. X. It will appear below that this criterion can probably be met, provided a single qualification is permitted. The qualification may be put in this way: the operating company is asked to guarantee that not more than 20 Kg. X of isotopic concentration greater than 5700005 weight per cent, is lost per year. Since the active cascade can not be separated into a section above 5700005 weight per cent X, and one below, it will still be necessary to run precise material balances around the whole cascade. However, the proposed relaxation will make it much easier to maintain material balances around the auxiliary plants (recovery and conversion, waste and feed storage, etc.), without adverse effect on national security.

Since material balances will be made each month, it will be possible to watch, and partly to control, the accumulated "unaccounted-for X", (called E_X throughout this report). By propagation-of-error methods, (given in full below) it will be possible to know the precision of the accumulated E_X, month by month. The upper limit, then, of $E_{X_A} \pm d(E_{X_A})$ is to be set at 20 Kg. The subscript A refers to the annual value. It is proposed to work at the "95 per-cent level of certainty" in the sense that we expect to exceed the allowable limit only one year in twenty, (and even then, not by much). This condition is expressed in symbols by Equation (1)

$$\bar{E}_{X_A} \pm d(0.10, E_{X_A}) = 10 \pm 10 \leq 20 \qquad (1)$$

In this way we require no more precision than is necessary to keep below the upper limit and at the same time permit ourselves to actually lose 10 Kg. X annually.

The precision given above as a confidence belt is now translated into a variance and the latter quantity is allocated to the different types of measurement that must be made in the precise "overall material balance on X", (abbreviated in this report as OMBX).

If a confidence belt is at the 90 per cent level of certainty then it will be exceeded on the high side, one time in twenty. Such a belt is roughly of width 1.7 standard deviations.

$$\text{If } 1.7\ s\ (E_{X_A}) = 10 \text{ Kg. X} \tag{2}$$

$$\text{Then } s^2\ (E_{X_A}) = \text{Variance } (E_{X_A}) = V\ (E_{X_A}) = 34.6 \text{ Kg.}^2\text{X} \tag{3}$$

This then, is the total amount of variance available for apportionment among all the errors that may be made in the annual OMBX. Table I outlines the apportionment proposed.

Table I

Variance Assigned to Sources of Uncertainty

in Annual Overall-Material Balance on X

Source of Uncertainty	Annual Variance Kg.^{2}X	Equivalent precision (95% confidence belt)	
		Kg.X/yr.	Per Cent
Consumption	22.3	± 9.4	± 4.0
Main Streams	9.4	± 6.1	± 0.13 (of feed)
Active Cascade Inventory Change	0.3	± 1.1	± 0.22
All other inventory	0.1	± 0.6	
Contingency	2.5	± 3.2	
Total	34.6		

A triple of overlapping criteria must be used to justify these assignments, (even after the maximum possible amount is left to cascade consumption, since this latter figure is so difficult to determine with precision), as follows:

a. The annual limit on E_{X_A} must be guaranteed.

b. Monthly material balances must be of sufficient precision to give some indication of the growth of E_X.

c. The overall material balance on T should be of sufficient precision to give some information on consumption and on the average X-concentration of the losses.

All three of these are covered by the assignments made.

Part II Assignment of Variance to Stream Measurements

The available variance due to streams is first divided into two equal parts, of 4.7 Kg.^{2} M, one part for random errors in daily measurements, the other for uncertainty in the constants used in computing the OMBX. The required precision per measurement, and the required number of measurements of each type is deduced here. A complete table of nomenclature is given below.

It is shown that better precisions than those required by the equal division of variance are already attainable. The variance saved is assigned to the constants whose target precisions are not yet attained.

Table II summarizes the results, in terms of per cent confidence belts (π).

TABLE II

Required Precisions of Stream Variables and Constants

Item	Variables: Measurement	Variables: Sampling	Constants
Feed - Weight	8 lbs/cylinder	--	6 lbs./cylinder
Purity	0.15%	0.15%	0.088%
Assay	--	--	0.22 %
Waste - Weight	8 lbs/cylinder	--	6 lbs/cylinder
Purity	0.15%	0.15%	0.088%
Assay (ratio)	0.40%	0.40%	0.097%
Product - Weight	1 gm/cylinder	--	0.64 gms/cylinder
Purity	0.15%	0.15%	0.176%
Assay	0.1 %	0.1 %	0.25 %
Feed minus Waste Weight	--	--	1 lb/cylinder

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A = Difference between the amount fed and removed from the cascade, kilograms
C = Consumption, kilograms
δ = Feed minus waste, pounds
E = Kilograms unaccounted-for
K = Ratio of kilograms uranium metal per pound of uranium hexafluoride $\doteq$ 0.307
$\mathcal{D}$ = Concentration of the consumed plus unaccounted-for material
p = Chemical purity, weight fraction
T = Uranium metal, kilograms
w = Net weight, pounds
x = Weight fraction of uranium 235 in uranium metal
X = Uranium 235, kilograms

Subscripts

A refers to a period of one year
b refers to bias
c refers to the cascade material balance envelope
D refers to a period of one day
F refers to feed material
m refers to a period of one month
P refers to product material
r refers to random
T refers to uranium metal
W refers to waste material
x refers to uranium metal

Prefixes

Read $\mathcal{L}(\beta, \alpha)$ as the confidence limits of α at the β level

Read $\underset{2,1}{\Delta}(\alpha)$ as the change in α

Read $n(\alpha)$ as the number of measurements of α

Read $\pi(\alpha)$ as the percent precision of α

Read $\sigma(\alpha)$ as the standard deviation of α

Read $V(\alpha)$ as the variance of α

The propagation-of-error equations for measured stream variables are given in Report 2.16.13 (E_q, A-35) and are used here without change. The cascade material balance equation written in the familiar form has*

$$E_{xA} = -\Delta_{2,1}(X_c) - C_{xA} + K\sum_{D=1}^{365}\left(\bar{p}_{FD}\,\bar{x}_{FD}\,w_{FD} - \bar{p}_{WD}\,\bar{x}_{WD}\,w_{WD} - \bar{p}_{PD}\,\bar{x}_{PD}\,w_{PD}\right) \quad (4)$$

It is a fundamental principle of the method used that the variables whose errors are to be propagated appear explicitly and only in one position in the equation being studied. It is further required that the observed variables appear in the equation. Now, the variables which are measured each day are not exactly the same as those whose systematic errors are used in the "propagation of bias" which is discussed below. In particular, random errors in isotopic concentrations occur only in daily measurements of $\bar{x}_{WD}$ and $\bar{x}_{PD}$, and not in x_F^i.

However, systematic error in $\bar{x}_W$ will be due to two independent factors. The first of these is the daily use of the same value for x_F^i which is, of course, only known with a certain precision. The second factor is the use of a waste-standard the ratio of whose concentration to the true value of normal material (x_W/x_F) is only known with a certain precision.

---

* The meaning of all symbols is the same as in the Report 2.16.13 except for p and w; the former refers here to chemical purity, i.e. pounds ^{235}U per pound of weighed material, and the latter to net weights in pounds.

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Similarly, a large part of the possible bias in weighing is eliminated by matching the daily feed to the daily waste on the scales of Section 630. Thus the equation for "propagation-of-bias" must reflect these facts even though Equation (4) is satisfactory for "propagation-of-random-errors". Equation (5) satisfies these conditions.

$$E_{XA} = -\underset{2,1}{\Delta}(X_C) - C_{XA}$$
$$+ K x_F \sum_{D=1}^{365} \left[\bar{p}_{FD}(\bar{w}_{FD} - \bar{w}_{WD}) + \bar{w}_{WD}\left(\bar{p}_{FD} - \bar{p}_{WD}\frac{\bar{x}_W}{\bar{x}_F}\right)\right]$$
$$+ K \sum_{D=1}^{365} \bar{p}_{PD}\,\bar{x}_{PD}\,\bar{w}_{PD} \qquad (5)$$

An expression for the variance due to random errors will be developed for the last three terms of Equation (4). Calling their sum A, applying the usual equation for the variance of a sum of statistically independent quantities, and assuming that the daily variance is roughly constant:

$$V_r(A_{XA}) = 365\,K^2\Big[(\bar{x}_F\bar{w}_{FD})^2 V_r(\bar{p}_{FD}) + (\bar{x}_W\bar{w}_{WD})^2 V_r(\bar{p}_{WD}) + (\bar{x}_P\bar{w}_{PD})^2 V_r(\bar{p}_{PD})$$
$$+ (\bar{p}_W\bar{w}_{WD})^2 V_r(\bar{x}_{WD}) + (\bar{p}_P\bar{w}_{PD})^2 V_r(\bar{x}_{PD})$$
$$(\bar{p}_F\bar{x}_F)^2 V_r(w_{FD}) + (\bar{p}_W\bar{x}_W)^2 V_r(w_{WD}) + (\bar{p}_P\bar{x}_P)^2 V_r(w_{PD})\Big]$$
$$(6)$$

It will be convenient to use percent confidence belts (π) in place of variances. The former are defined for any quantity Z by the relation:

$$V(Z) = \left(\frac{Z}{200}\right)^2 \pi_+^2(Z) \tag{7}$$

Substituting π_+'s for V_+'s in Equation (6), and placing all p's in coefficients equal to 1, we have:

$$V_+(P_{WD}) = 365\left(\frac{K}{200}\right)^2 \left[(x_p \bar{w}_{PD})^2 \frac{\pi_+^2(p_P)}{n(p_{PD})} + (\bar{x}_W \bar{w}_{WD})^2 \frac{\pi_+^2(p_W)}{n(p_{WD})} + (x_p w_{PD})^2 \frac{\pi_+^2(p_P)}{n(p_{PD})} \right.$$

$$+ (\bar{x}_W \bar{w}_{WD})^2 \frac{\pi_+^2(x_W)}{n(x_{WD})} + (\bar{x}_p \bar{w}_{PD})^2 \frac{\pi_+^2(x_P)}{n(x_{PD})}$$

$$\left. + (x_p \bar{w}_{PD})^2 \frac{\pi_+^2(w_P)}{n(w_{PD})} + (\bar{x}_W \bar{w}_{WD})^2 \frac{\pi_+^2(w_W)}{n(w_{WD})} + (x_p w_{PD})^2 \frac{\pi_+^2(w_P)}{n(w_{PD})} \right] \tag{8}$$

The coefficients of all terms in this equation can be estimated from current plant data. We find:

$$V_T(A_{KA}) = 0.81\,\frac{\pi_T^2(\rho_F)}{\lambda(\rho_{FD})} + 0.39\,\frac{\pi_T^2(\rho_W)}{\lambda(\rho_{WD})} + 0.058\,\frac{\pi_T^2(\rho_P)}{\lambda(\rho_{PD})}$$

$$+ 0.39\,\frac{\pi_T^2(x_W)}{\lambda(x_{WD})} + 0.058\,\frac{\pi_T^2(x_P)}{\lambda(x_{PD})}$$

$$+ 0.81\,\frac{\pi_T^2(w_F)}{\lambda(w_{FD})} + 0.39\,\frac{\pi_T^2(w_W)}{\lambda(w_{WD})} + 0.058\,\frac{\pi_T^2(w_P)}{\lambda(w_{PD})} \qquad (9.)$$

If we assume now that the precision of purity analyses is about the same in feed, waste and product*, and that each of the other types of measure should contribute equally to the total allowed variance of 4.7, we find that the current precisions are better than the required precisions in all cases. It will be possible then, to save some variance by using the available precisions. These precisions are indicated in Table II, under "variables". An equal amount of variability, expressed as a precision has been allowed for sampling errors in each type of measurement on each stream. In this way 4.4 Kg.^{2} X units of variance are made available. This quantity is distributed over the six purity-and-assay constants as will be explained below.

---

* Even if product purity is only measured with half the precision of feed and waste, the uncertainty in the annual value is negligible.

The error in the annual material balance due to unknown biases in the stream properties accumulates at a different rate from that due to random errors. Thus, the variance of a sum of daily measurements due to random errors may be expressed by the equation

$$V_r\left(\sum_{D=1}^{N_D} A_D\right) = \sum_{D=1}^{N_D} V_r(A_D) = (N_D)\ V_r(A_D) \qquad (10.)$$

But the variance of a sum of daily values, due to systematic errors will accumulate more rapidly since the same errors are made in the same direction each day. Thus,

$$V_b\left(\sum_{D=1}^{N_D} A_D\right) = V_b(N_D A_D) = N_D^2\, V_b(A_D) \qquad (11.)$$

In the above equations, A_D is any quantity based on daily measurements, and N_D is the number of days. Equation (12) below is derived from Equation (5), and not from Equation (4), since the former equation represents adequately the variables in which systematic errors are certainly made. The same (erroneous) value of x_F^1 is used every day, the value of x_W is biased every day because the waste standard is given a constant erroneous value, and similar considerations hold for the other quantities appearing in Equation (2). It will be noted that the annual variance due to unknown bias is $(365)^2$ times the daily value, while the annual variance due to random errors in daily measurements is only 365 times the random daily variance.

$$V_b(A_{vA}) = (365K)^2 V_b\left[x_F'\left\{\bar{p}_{FD}\bar{\delta}_D + \bar{w}_{WD}\left(\bar{p}_{FD} - \bar{p}_{WD}\frac{\bar{x}_{WD}}{x_F'}\right)\right\} - \bar{p}_{PD}\bar{x}_{PD}\bar{w}_{PD}\right]$$

$$= (365K x_F')^2\Big\{\bar{p}_F^{\,2} V_b(\delta_D) + \bar{\delta}_D^{\,2} V_b(p_{FD}) + (\bar{w}_{WD})^2 V_b\left(p_{FD} - p_{WD}\frac{x_{WD}}{x_F'}\right)$$

$$+ \left(\bar{p}_F - \bar{p}_W\frac{\bar{x}_W}{x_F'}\right)^2 V_b(w_{WD}) + \left[\bar{p}_F\bar{\delta}_D + \bar{w}_{WD}\left(\bar{p}_F - \bar{p}_W\frac{\bar{x}_W}{x_F'}\right)\right]^2 \frac{V_b(x_F')}{x_F'^{\,2}}$$

$$+ \left(\frac{\bar{p}_P\bar{x}_P\bar{w}_{PD}}{x_F'}\right)^2\left[\frac{V_b(p_P)}{(\bar{p}_P)^2} + \frac{V_b(x_P)}{(\bar{x}_P)^2} + \frac{V_b(w_{PD})}{(\bar{w}_{PD})^2}\right]$$

$$= (365K x_F')^2\Big\{\bar{p}_F^{\,2}\bar{\delta}_D^{\,2}\left[\frac{V_b(\delta_D)}{(\bar{\delta}_D)^2} + \frac{V_b(p_F)}{(\bar{p}_F)^2}\right]$$

$$+ (\bar{w}_{WD})^2\left[V_b(p_F) + \left(\frac{\bar{x}_W}{x_F'}\right)^2 V_b(p_W) + (\bar{p}_W)^2 V_b\left(\frac{x_W}{x_F}\right)\right]$$

$$+ \left(\bar{p}_F - \bar{p}_W\frac{\bar{x}_W}{x_F'}\right)^2 V_b(w_{WD}) + \left(\bar{p}_F\bar{w}_{FD} - \bar{p}_W\bar{w}_{WD}\frac{\bar{x}_W}{x_F'}\right)^2 \frac{V_b(x_F')}{x_F'^{\,2}}$$

$$+ \left(\bar{p}_P\frac{\bar{x}_P}{x_F'}\bar{w}_{PD}\right)^2\left[\frac{V_b(p_P)}{(\bar{p}_P)^2} + \frac{V_b(x_P)}{(\bar{x}_P)^2} + \frac{V_b(w_{PD})}{(\bar{w}_{PD})^2}\right]\Big\}$$

(12)

There are two terms in V_b (p_F) in this equation; the first is obviously negligible with respect to the second and is, therefore, omitted below. Substituting π_b's for V_b's in Equation (12), and placing all p's in coefficients equal to 1, we have:

$$V_b(A_{xn}) = \left(\frac{365\ K\ \chi_F'\ \bar{\omega}_{WO}}{200}\right)^2 \Bigg[\pi_b^2(\rho_F) + \left(\frac{\bar{\chi}_W}{\chi_F'}\right)^2 \pi_b^2(\rho_W) + \left(\frac{\bar{\chi}_P \bar{\omega}_{PO}}{\chi_F' \bar{\omega}_{WO}}\right)^2 \pi_b^2(\rho_P)$$

$$+ \left(1 - \frac{\bar{\chi}_W}{\chi_F'}\right)^2 \pi_b^2(\chi_F') + \left(\frac{\bar{\chi}_W}{\chi_F'}\right)^2 \pi_b^2\left(\frac{\chi_W}{\chi_F}\right) + \left(\frac{\bar{\chi}_P \bar{\omega}_{PO}}{\chi_F' \bar{\omega}_{WO}}\right)^2 \pi_b^2(\chi_P)$$

$$+ \left(\frac{\bar{\delta}_O}{\bar{\omega}_{WO}}\right)^2 \pi_b^2(\delta_O) + \left(1 - \frac{\bar{\chi}_W}{\chi_F'}\right)^2 \pi_b^2(\omega_{WO}) + \left(\frac{\bar{\chi}_P \bar{\omega}_{PO}}{\chi_F' \bar{\omega}_{WO}}\right)^2 \pi_b^2(\omega_{PO}) \Bigg] \tag{13}$$

The equation with the coefficients evaluated follows:

$$\begin{aligned} V_b(A_{xn}) = \ & 290\ \pi_b^2(\rho_F) + 140\ \pi_b^2(\rho_W) + 21\ \pi_b^2(\rho_P) \\ & + 28\ \pi_b^2(\chi_F') + 140\ \pi_b^2\left(\frac{\chi_W}{\chi_F}\right) + 21\ \pi_b^2(\chi_P) \\ & + 0.03\ \pi_b^2(\delta_O) + 28\ \pi_b^2(\omega_{WO}) + 21\ \pi_b^2(\omega_{PO}) \end{aligned} \tag{14}$$

The total variance available for the unknown biases in these mine stream factors is 4.7 + 4.4, or 9.1 Kg.^{2} X per year. The 4.7 units represent the original allocation of half the 9.4 units permitted by Table I. The 4.4 units represent the amount not needed by random variables since the existing precision-due-to-random-error of all the daily measures appears to be better than good. We can make good estimates of the last three terms in Equation (14) since it is not difficult to set upper limits on the possible uncorrected bias in weighings. We put, therefore,

$$\pi_b(\delta) = \pm 2.3\% \text{ or } \pm 1 \text{ lb.}$$

$$\pi_b(W_{wD}) = \pm 0.14\% \text{ or } \pm 6 \text{ lbs.}$$

$$\pi_b(W_{pD}) = \pm 0.14\% \text{ or } \pm 0.64 \text{ gm.}$$

These three assumptions, substituted into Equation (14) give 1.12 Kg.^{2} X units of variance. We are left then with 9.1 - 1.1 or 8.0 units, which we arbitrarily allocate equally between the purity variables and the assay variables. It will be fair to assume that the (unknown) bias in the method used to measure purity of product will be twice as large as that in the method used for feed and waste analyses, since it will not be possible to use large samples, or as many analyses, in determining the former. For the three assay methods, however, it seems reasonable, as a first approximation, to allow the same contribution to annual variance for each. We have then:

$$290\,\pi_b^2(\rho_F) + 140\,\pi_b^2(\rho_F) + 21 \times 4\,\pi_b^2(\rho_F) = 4.0$$

$$514\,\pi_b^2(\rho_F) = 4.0$$

$$\pi_b(\rho_F) = \pi_b(\rho_W) = \pm 0.088\% \quad ; \quad \pi_b(\rho_P) = \pm 2 \cdot 0.088\% = 0.176\%$$

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Also: $28\,\pi_b^2(x_F') = 140\,\pi_b^2\left(\frac{x_W}{x_F'}\right) = 21\,\pi_b^2(x_P) = 4.0/3 = 1.33$

Whence

$$\pi_b\,(x_F') \cong \pm 0.22$$

$$\pi_b\,(x_W/x_F') \cong \pm 0.097$$

$$\pi_b\,(x_P) \cong \pm 0.25$$

These target values are all shown in numerical substitution in Equation (14a), below, and are also entered in Table II.

$$\begin{aligned} 9.1 \cong\ & 290\,(0.088)^2 + 140\,(0.038)^2 + 21\,(0.176)^2 \\ & + 28\,(0.22)^2 + 140\,(0.10)^2 + 21\,(0.25)^2 \\ & + 0.03\,(2.3)^2 + 28\,(0.14)^2 + 21\,(0.14)^2 \\ \cong\ & 2.2 + 1.2 + 0.6 \\ & + 1.4 + 1.4 + 1.3 \\ & + 0.1 + 0.5 + 0.4 \end{aligned} \qquad (14A)$$

It has been assumed that there is no correlation between the systematic errors in the three chemical purities measured. This is plausible since the impurities in each stream are different and these are thought to influence the systematic error. It has been assumed further that as they are now being determined, there is no correlation between the systematic errors in the measurements of the standards x_F', x_W/x_F' and x_P. Finally it is thought reasonable to assume no correlation between the systematic error in the difference between feed and waste weights (δ_0) and the waste weight (ω_{WD}).

Allowance can easily be made later for changes in the schedule of number-of-T-analyses, number of K-assays, and number-of-weighings by means of the equations given. Similarly, changes in operating conditions will reflect in changed coefficients as shown by the same equations. It is interesting to note that there is no considerable change in the required precisions of the assay standards from those given in Report No. 2.16.13. The reasons for this are to be found in the facts that the bottom concentration, feed rate, and daily X draw-off have not changed appreciably. The increase in consumption-rate shows up as a slight decrease in the required precision of the x_p standard.

The effect of all the precisions of Table II on the precision of the annual overall material balance of T (OMBT) will now be given. The propagation of random errors in stream-variables is carried through in exact analogy to Equations (4) through (8). Alternatively Equation (15) can be obtained from Equation (8) by placing all x's in the latter equal to 1, and all V(x) terms equal to zero.

$$V_t(A_{TA}) = 365\left(\frac{K}{200}\right)^2\left[(\bar{w}_{FD})^2\frac{\pi_T^2(\rho_F)}{n(\rho_{FD})} + (\bar{w}_{WD})^2\frac{\pi_T^2(\rho_W)}{n(\rho_{WD})} + (\bar{w}_{PD})^2\frac{\pi_T^2(\rho_P)}{n(\rho_{PD})}\right.$$

$$\left. + (\bar{w}_{FD})^2\frac{\pi_T^2(w_F)}{n(w_{FD})} + (\bar{w}_{WD})^2\frac{\pi_T^2(w_W)}{n(w_{WD})} + (\bar{w}_{PD})^2\frac{\pi_T^2(w_P)}{n(w_{PD})}\right] \qquad (15)$$

It is seen that the contribution of the product stream to variance is negligible. Assuming the values of π_b's specified in Table II, we find that

$$V_b(A_{TA}) = 15{,}400\left[2(0.15)^2 + 2(0.15)^2 + (0.185)^2 + (0.185)^2\right]$$

$$= 15{,}400 \times 0.158 = 2{,}400 \text{ Kg.}^2 \text{ T.} \qquad (16)$$

Similarly for the uncertainty in CMBT due to biases in the stream constants:

$$V_b(A_{TA}) = \left(\frac{365K}{200}\right)^2\left[(\bar{w}_{FD})^2\pi_b^2(\rho_F) + (\bar{w}_{WD})^2\pi_b^2(\rho_W) + (\bar{w}_{PD})\pi_b^2(\rho_P) + (\bar{\delta}_D)^2\pi_b^2(\delta) + (\bar{w}_{PD})^2\pi_b^2(w_P)\right]$$

$$= 83{,}600 \text{ Kg.}^2 \text{T.} \qquad (17)$$

If the daily consumption of T is assumed to be 10.7 ± 4%, then an estimate of the variance of the OMBT can be made.

$$V(E_{TA}) = V(\underset{2,1}{\Delta} T_c) + V(C_{TA}) + V_r(A_{TA}) + V_b(A_{TA})$$

$$= 225 + \left(\frac{10.7 \times 0.04 \times 365}{2}\right)^2 + 2,400 + 83,600$$

$$= 92,000 \ Kg.^2 \ \bar{X} \qquad (18)$$

$$Q(0.05, E_{TA}) = \pm 2\sqrt{92,000} = \pm 610 \ Kg. \ T. \qquad (19)$$

The OMBT and OMBX may be used to estimate consumption and the average concentration of consumed material. Equation (4) and its equivalent for the OMBT can be rearranged with E_A and C_A on the left hand side. If we are willing to assume that E_A is close to zero, we may then solve for the corrosion term.

$$V(E_{XA} + C_{XA}) = V(\underset{2,1}{\Delta} \bar{X}_c) + V_r(A_{XA}) + V_b(A_{XA}) \qquad (20)$$

$$= 12.3$$

$$Q(0.05, E_{XA} + C_{XA}) = \pm 2\sqrt{12.3} = \pm 7 Kg. \ \bar{X} \qquad (21)$$

≅ ±3% of annual consumption

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$$V(E_{TA} + C_{TA}) = V(\underset{\Delta,V}{\Delta}, T_c) + V_-(A_{TA}) + V_+(A_{TA}) \qquad (22)$$

$$= 225 + 2{,}400 + 83{,}600$$

$$= 86{,}000 \text{ kg}^2\text{T}$$

$$Q(0.05, E_{TA} + C_{TA}) = \pm 2\sqrt{86{,}000} = \pm 590 \text{ kg.T.} \qquad (23)$$

$$\doteq \pm 15\% \text{ of annual consumption}$$

The ratio $J = \dfrac{E_{XA} + C_{XA}}{E_{TA} + C_{TA}}$ is approximately the average concentration of the consumed material.

$$V(J) = J^2 \left[\frac{V(E_{XA} + C_{XA})}{(E_{XA} + C_{XA})^2} + \frac{V(E_{TA} + C_{TA})}{(E_{TA} + C_{TA})^2} \right] \qquad (24)$$

$$\doteq (0.064)^2 \left[\frac{12.3}{(250)^2} + \frac{86{,}000}{(3{,}900)^2} \right]$$

$$= 25 \times 10^{-6}$$

$$Q(0.05, J) = \pm 2\sqrt{25 \times 10^{-6}} = \pm 0.01 \qquad (25)$$

Thus the average concentration of the annually consumed material can be found by material balances to about ± 1 weight per cent.

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Recommendations and Conclusions

1. The precisions of analyses, of assays, and of weighings are all now at acceptable levels, and only some continuing guarantee of their stability in the form of quality control is necessary.

2. The target accuracies (precision of constants, or of methods of measurement) given by Table II are essentially unchanged from the Report No. 2.16.13. Their stability can only be guaranteed by freezing methods of analyses, assay and weighing.

3. The stream conditions required to meet the guarantee of "not more than 20 Kg. of X unaccounted-for per year" are given in Table II.

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